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# Shooting approach in optimized boundary value orbit determination

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**Abstract**

Space debris is becoming a major threat for functional spacecraft. The debris population needs to be monitored and catalogued to better characterize the space environment. Optical surveys is one mean to get information about its population distribution. The observations acquired in optical surveys are sparse and cover a very small part of the orbit, hence the initial orbit determination becomes challenging. Commonly, two observation series are associated together to find out if they belong to the same object and an initial orbit is computed. The latter can be performed using the Optimized Boundary Value Initial Orbit Determination (OBVIOD) approach, which is an existing method to associate short-arc angles-only observations. In the original version of this method, the initial orbit determination takes place by solving the Lambert's problem with a model assuming pure Keplerian orbits without including perturbations. In this work, to include the latter we use a so-called shooting scheme. This approach takes a hypothesis at the initial boundary and propagates it to the second boundary, where the computed value and the original boundary value are compared. The hypothesis, which gives the desired output at the second boundary, is accepted as the solution. In the proposed algorithm, the propagation from the initial boundary to the final one involves perturbations such as solar radiation pressure, Earth's geopotential terms, solar and lunar gravitational forces. A root-finding Newton method is used inside the shooting iteration. An additional difficulty in the proposed algorithm arises in multi-revolutions scenarios, where multiple solutions of the Lambert problem are possible. Tests were done using simulated short-arc angles-only observations, separated by single or multiple revolutions, and different area-to-mass ratio values for the observed objects. The performance of the orbit determination procedure is evaluated in the different scenarios.

## 1 Introduction

The increasing population of space debris poses a serious threat to spacecraft in orbit. Therefore monitoring of the space objects population is of fundamental importance to prevent risks in the space environment. Monitoring can be performed with passive optical surveys, in which the observations are sparse and cover a very small part of the orbit. This makes the initial orbit determination out of these observations difficult. The sparse and short sequences of observations are called tracklets and have to be correlated in a process called tracklet correlation [RD-1]. Different methods were proposed to approach the correlation problem. In general, a hypothetical orbit in common for two tracklets is calculated. If the orbit matches the observations, it is assumed that the tracklets are associated to the same object and the correlation is positive. The methods might use an initial value or a boundary value formulation. The solution space of the problem can be restricted to a certain admissible region and the observations are usually reduced to so-called attributables [RD-2].

In the methods based on a boundary value scheme the range hypotheses at two epochs are taken to calculate an orbit common to the two attributables and the matching is evaluated to accept the correlation. Here, the ranges together with the angles characterize a Lambert's problem. The solution in the range space can be calculated e.g. with an optimization scheme [RD-3]. This approach is referred as Optimized Boundary Value Initial Orbit Determination (OBVIOD) method. Then the initial orbit calculated using the correlation algorithm is commonly improved with e.g. a batch least squares procedure making use of all the available observations.

In the present paper we want to extend the OBVIOD formulation, since the Lambert solution takes into account only pure Keplerian orbits and not additional perturbations. One way to consider perturbations is the shooting method [RD-5]. In the latter one iterates over possible initial values at the first boundary and check the propagated values with the boundary conditions at the second epoch. The initial value that better matches the boundary is the selected solution. In the propagation the Earth's geopotential terms, lunisolar gravitational forces, and solar radiation pressure (SRP) are treated. In a geostationary regime the SRP forces start to be relevant especially after several orbital revolutions and if the objects have a high area-to-mass ratio (AMR). The shooting scheme can be solved with a Newton algorithm, or e.g. a bisection approach [RD-4], while the optimization in the range space can be formulated as a least squares problem or, as in the original work, applying a Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm [RD-5]. Moreover, the shooting arc can be divided into shorter intervals in a multiple shooting procedure to improve the algorithm performance. In this work we will consider only the combination of single

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shooting with Newton and least squares algorithms. The analysis will address the accuracy dependence of the proposed method from the arc length between the tracklets and the AMR values.

## 2 Optimized boundary value method

Tracklets are short-arc series of topocentric measurements in right ascension $\alpha$ and declination $\delta$, from several seconds to few minutes long. The attributable is a reduced form of a tracklet at the epoch $t$:

$$\mathbf{A}_t = \left(\alpha, \dot{\alpha}, \delta, \dot{\delta}\right)_t \tag{1}$$

where the angular rates are computed through a linear regression. The boundary value problem is expressed with the angular positions $\alpha_{1,2}, \delta_{1,2}$ at the boundary epochs $t_1, t_2$ and with hypothetical topocentric ranges $\rho_1, \rho_2$. From these parameters the related orbit, and thus the angular rates $\dot{\alpha}'_{1,2}, \dot{\delta}'_{1,2}$, can be computed with a Lambert solver [RD-6]. The difference between the computed $\dot{\alpha}'_{1,2}, \dot{\delta}'_{1,2}$ and the observed rates $\dot{\alpha}_{1,2}, \dot{\delta}_{1,2}$ is expressed as Mahalanobis distance:

$$d_{\mathrm{M}} = \sqrt{(\dot{\boldsymbol{z}} - \dot{\boldsymbol{z}}')^{\mathrm{T}} \mathbf{C}^{-1} (\dot{\boldsymbol{z}} - \dot{\boldsymbol{z}}')} \tag{2}$$

where $\dot{\boldsymbol{z}} = \left(\dot{\alpha}_1, \dot{\delta}_1, \dot{\alpha}_2, \dot{\delta}_2\right)$, $\dot{\boldsymbol{z}}' = \left(\dot{\alpha}'_1, \dot{\delta}'_1, \dot{\alpha}'_2, \dot{\delta}'_2\right)$, and $\mathbf{C} = \mathbf{C}_{\dot{z}} + \mathbf{C}_{\dot{z}'}$ is the sum of the covariance matrices related to $\dot{\boldsymbol{z}}$ and $\dot{\boldsymbol{z}}'$. The solution of the orbital problem is searched minimizing the distance function $d_{\mathrm{M}}$ in the space $\rho_1, \rho_2$. If the latter is smaller than a given threshold, the orbit is compatible with both tracklets, and the correlation accepted.

## 3 Shooting method

The equations of motion of the orbital problem can be written as first-order differential equations

$$\dot{\mathbf{x}}(t) = \mathbf{f}(t, \mathbf{x}(t), \boldsymbol{\lambda}) \tag{3}$$

where $\mathbf{x}(t)$ is the state vector with positions and velocities, and $\boldsymbol{\lambda}$ are optional parameters. Given an initial value $\mathbf{x}(t_1) = \mathbf{x}_1$ we can integrate Eq. (3) and compute $\mathbf{x}(t_2) = \mathbf{x}_2$ at a later time $t_2$. The initial value $\mathbf{x}_1$ is the variable to be optimized to fulfil the condition at $t_2$. In our case the position vectors $\mathbf{r}_1(t_1)$ and $\mathbf{r}_2(t_2)$ define the boundary conditions and the shooting iteration searches for the appropriate initial velocity vector $\mathbf{v}_1(t_1)$ that fulfils the condition at $t_2$. We define explicitly $\mathbf{x} = (x, y, z, \dot{x}, \dot{y}, \dot{z})$ and the constraints

$$\mathbf{F}(\mathbf{x}) = \begin{pmatrix} x \\ y \\ z \end{pmatrix}(t_2) - \mathbf{r}_2(t_2) = 0. \tag{4}$$

The partial derivatives w.r.t. to the initial velocities are

$$\left(\frac{\partial \mathbf{F}(\mathbf{x})}{\partial \mathbf{x}}\right) = \begin{bmatrix} \frac{\partial x(t_2)}{\partial \dot{x}(t_1)} & \frac{\partial x(t_2)}{\partial \dot{y}(t_1)} & \frac{\partial x(t_2)}{\partial \dot{z}(t_1)} \\ \frac{\partial y(t_2)}{\partial \dot{x}(t_1)} & \frac{\partial y(t_2)}{\partial \dot{y}(t_1)} & \frac{\partial y(t_2)}{\partial \dot{z}(t_1)} \\ \frac{\partial z(t_2)}{\partial \dot{x}(t_1)} & \frac{\partial z(t_2)}{\partial \dot{y}(t_1)} & \frac{\partial z(t_2)}{\partial \dot{z}(t_1)} \end{bmatrix} \tag{5}$$

To find the solution $\mathbf{x}_0$ the following Newton iteration scheme is applied:

$$\mathbf{x}^{j+1} = \mathbf{x}^j - \left(\frac{\partial \mathbf{F}(\mathbf{x}^j)}{\partial \mathbf{x}^j}\right)^{-1} \mathbf{F}(\mathbf{x}^j) \tag{6}$$

## 4 Boundary value problem with shooting

In the proposed algorithm, the optimized search in the $(\rho_1, \rho_2)$ space is performed with a least squares approach and the Lambert solver is replaced by a shooting iteration. The least squares problem was addressed with a Gauss-Newton algorithm. Since the resolution matrix is often nearly singular, a Singular Value Decomposition was opted for its inversion. The whole structure consists of a shooting iteration within an optimization iteration (Figure 1):

- The shooting iteration goes over the initial velocity given the boundaries $\rho_1, \alpha_1, \delta_1$ and $\rho_2, \alpha_2, \delta_2$.
- The optimization iteration goes over $\rho_1$ and $\rho_2$ given $\dot{\alpha}_1, \dot{\delta}_1$ and $\dot{\alpha}_2, \dot{\delta}_2$ as discriminators in Eq. (2).

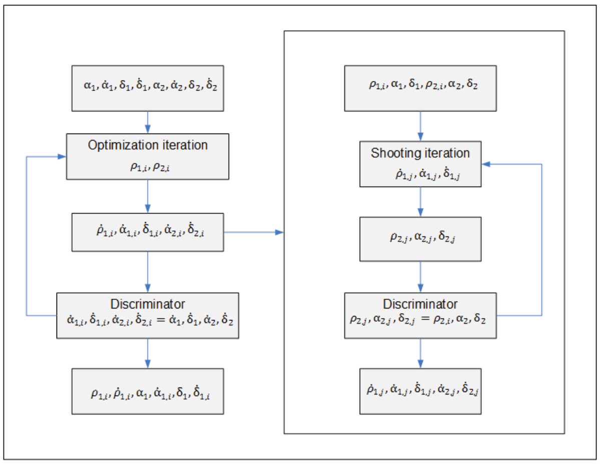

Figure 1: Scheme of algorithm.

The shooting structure for the Lambert problem takes position vectors $\mathbf{r}_1(t_1)$ and $\mathbf{r}_2(t_2)$ and loops over the initial velocity $\mathbf{v}_1(t_1)$. The positions $\alpha_{1,2}, \delta_{1,2}$ at the epochs $t_1, t_2$ and the topocentric ranges $\rho_1, \rho_2$ are transformed to the geocentric vectors $\boldsymbol{r}_1, \boldsymbol{r}_2$ required by the Lambert problem. The initial velocity is related to $\dot{\rho}_1, \dot{\alpha}_1, \dot{\delta}_1$ and the solution has to match the boundaries $\rho_1, \alpha_1, \delta_1$ and $\rho_2, \alpha_2, \delta_2$. As initial guess for $\dot{\rho}_1, \dot{\alpha}_1, \dot{\delta}_1$ in the shooting procedure the values provided by the unperturbed Lambert solver [RD-6] are taken.
In general, there are multiple solutions to the Lambert

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problem. Even if the prograde/retrograde sense of motion and the number of orbital revolutions are known, two solutions are possible. This forces to identify the correct solution at any optimization iteration step. Empirically, we can choose the solution with values closer to the measured angular rates, assuming that this is more likely to be the correct one. Based on the obtained results this seems to be a valid procedure in those cases where the two solutions are not too close. The latter situation depends on the geometry of the Lambert problem, specifically on the interval between the tracklets. In the scenarios simulated in this work no problematic convergence behavior was identified.

## 5 Results

The proposed method was applied to simulated observations in the geostationary region. We assume tracklets of 5 measurements separated by 30 s and astrometric accuracy around 0.5″. Two observed fields separated by 15° are simulated: objects observed in the first field at the orbital perigee are observed later again in a field shifted of 15° in right ascension. The second observation is generated after one, two, and three orbit revolutions. In the propagation, a simulated orbit with 0.01 eccentricity, lunisolar perturbation, geopotential coefficients up to degree and order 4, and Solar Radiation Pressure (SRP) were taken into account. The implementation in the Orekit library [RD-7] was adopted. In the SRP perturbation a simple model with isotropic radiation and reflection coefficient equal to 1.0 was assumed. Different values of area-to-mass ratio (AMR) were taken into account ranging from 0.1 to 10.0 $m^2$/kg.
The error of the determined orbit is assessed as the difference in position and velocity w.r.t. the simulated orbit, represented in radial, along-track, and cross-track (RSW) components. Table 2 shows the errors for different numbers of revolutions and AMR values. The norm of the position and velocity error vectors are marked in bold. Since the considered tracklets are short, typical position uncertainties of several kilometres in the computation of the initial orbit determination are expected, in agreement with the literature [RD-8]. Apparently the number of revolutions between the tracklets does not significantly affect the accuracy, given that the geometric arc length of 15° between the fields remains constant. The position inaccuracy is especially big in the radial component, possibly since angles and not range measurements are available. Differently, the along-track component of the velocity has the largest values, possibly coming from the derived angular rates. The error in position and velocity remains quite similar between 12’100 and 12’500 m, and between 1.82 and 1.84 m/s, respectively. The increase in AMR also does not essentially affect the accuracy. These relatively low errors indicate that the resulting orbit can be used, without incurring convergence problems, as starting point to perform an orbit improvement using the single measurements in a batch least squares procedure. For AMR values of 10.0 $m^2$/kg, the final orbit has errors around 2500 m and 1.2 m/s in position and velocity, respectively, i.e. the accuracy is acceptable to pursue follow-up observations and the build-up of an orbital catalogue.

| AMR [$m^2$/kg] | 1 rev. | | 2 rev. | | 3 rev. | |
|---|---|---|---|---|---|---|
| | Pos. [m] | Vel. [m/s] | Pos. [m] | Vel. [m/s] | Pos. [m] | Vel. [m/s] |
| 0.1 | 12’066 | 0.12 | 12’150 | 0.13 | 12’175 | 0.13 |
| | 178 | 1.81 | 179 | 1.81 | 179 | 1.81 |
| | 1’447 | 0.01 | 1’458 | 0.02 | 1’461 | 0.02 |
| | **12’154** | **1.82** | **12’238** | **1.82** | **12’264** | **1.82** |
| 1.0 | 12’068 | 0.12 | 12’156 | 0.11 | 12’185 | 0.12 |
| | 178 | 1.81 | 179 | 1.81 | 179 | 1.82 |
| | 1’447 | 0.01 | 1’459 | 0.02 | 1’462 | 0.02 |
| | **12’156** | **1.82** | **12’244** | **1.82** | **12’275** | **1.82** |
| 10.0 | 12’118 | 0.09 | 12’251 | 0.09 | 12’333 | 0.08 |
| | 179 | 1.82 | 180 | 1.83 | 181 | 1.84 |
| | 1’454 | 0.01 | 1’470 | 0.01 | 1’480 | 0.01 |
| | **12’207** | **1.83** | **12’341** | **1.83** | **12’423** | **1.84** |

Table 1. Orbital errors for different AMR values and numbers of revolutions. The RSW components and in bold the norm of the vectors are indicated.

## 6 Conclusions

We extended the OBVIOD method to include orbital perturbations using a shooting scheme. In the latter the perturbations are included in the numerical propagation of the orbit and the search of the initial orbital values is performed with a multi-variable Newton algorithm. The optimization problem searching for the boundary ranges is addressed with a least squares algorithm. A Lambert solver provides the starting values of the shooting routine. Due to the implicit solution ambiguity in the Lambert problem, a discrimination based on the measured angular rates was introduced: the constraint revealed to be suitable for the simulated observation geometry.
In the simulated observation scenarios different values of arc length in the geostationary ring and area-to-mass ratio values were assumed.
The results show that for solar radiation pressure perturbations during up to three orbit revolutions and assuming AMR values up to 10 $m^2$/kg, the proposed method produces an orbit accurate enough to be used as a priori state in a least squares orbit improvement based on the single measurements. The refined orbits supposedly allow for further follow-up observations and ultimately for catalogue build-up. Still, other evaluations

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of the method are necessary with e.g. different observation scenarios, orbital shape, measurement noise, or objects in another orbital regime. Moreover, the performance of the method using real observations will need to be investigated in future work.

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