MULTI-SENSOR SPACE OBJECT TRACKING FOR TUMBLING MOTION CHARACTERIZATION

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Abstract

The knowledge of the attitude dynamics of passive space objects or decommissioned satellites gains importance in the rapidly growing sector of Close-Proximity Operations (CPO) for In-Orbit Servicing or Active Debris Removal. In particular, knowing the spin axis orientation, the spin period and the rate of change of such parameters in the reference frame of choice is necessary for the optimum decision on the method that will be used to perform the in-orbit operation safely and efficiently.

In this paper we report the efforts and the results obtained in a study supported by ESA which aims at the development of methods for the determination of the spacecraft attitude motion and its evolution. Multiple detection technologies were operated during the project, including Satellite and Space Debris Laser Ranging, CCD and Single Photon Avalanche Diode (SPAD) photon-counter light curves, as well as measurements from tracking and imaging radars. The data from these observation techniques can be exploited in a complementary way through two different approaches for attitude determination known as “amplitude” and “epoch” methods. CCD photometric measurements are more suited to the former method, which is based on the amplitude of the intensity variations in the light curve. On the other hand, laser ranging data and single-photon counter light curves better fulfil the requirements for the temporal analysis in the epoch method, which extracts the tumbling parameters from the embedded temporal signals and benefits from the difference between synodic and sidereal rotation rate of the object. In addition to the combination of these two approaches, radar measurements of selected objects-of-interest (OOI) were performed for validation during a joint tracking campaign.

Models for the attitude evolution were analysed and simulated using the In-Orbit Tumbling Analysis (ιOTA) tool, which was further improved and validated in the current project. The comparison of the propagated attitude with the one determined from measurements serves as validation for the attitude determination methods, the simulator, and the evolution models.

1 Introduction

One of the solutions proposed to face the growing problem of orbital debris is active debris removal (ADR). This approach has to be followed to remove large-mass objects like disposed upper stages or big satellites, and stabilize or better reduce the future debris population. The knowledge of the attitude state of the candidate space object is crucial for the mission and for the choice of the removing approach. The same applies in contingency situations, where the affected satellite is not responsive any longer and any information, included the attitude state, can be useful to understand the problem. While for operational satellites the attitude state can be deduced from telemetry data, for debris objects different observation techniques like photometric measurements, laser ranging, and radar data are used.

In photometric measurements the variation of the brightness due to reflected sunlight of the object over time, also called light curve, provides information about the rotation of the observed object. Several studies were performed to extract debris rotation rates from light curves [1][2][3]. The combination of light curves and Satellite Laser Ranging (SLR) data was used to derive the spin period and analyze the temporal evolution of space debris [4][5]. Kirchner et al. [7] combined so-called single-photon light curves and SLR measurements for the same purpose. In [6] multiple methods (passive optical, SLR, radar) are used for validation and refinement of the attitude state. The spin axis orientation of defunct satellites equipped with retroreflectors was identified in several works [8][9][10]. Santoni et al. [12] determined from light curves the direction of the rotation axis of a rocket body (R/B). Their approach, also called “amplitude method” is based on the ratio of the maximal and minimal brightness extracted from the light curve. In [13] with a similar approach, the authors were able to estimate in

addition to the spin axis of a rocket body also its precession motion. A different approach for attitude determination is the so-called “epoch method” described in [14][15], which extracts the tumbling parameters from the embedded temporal signals and benefits from the difference between synodic and sidereal rotation rate of the object. Recently, Zhao et al. were able to deduce the attitude parameters of a rocket body combining the epoch method and a procedure based on fitting residuals from light curves and SLR measurements [11].
In this paper we report the efforts and the results obtained in a study supported by ESA which aims at the development of methods for the determination of the spacecraft attitude motion and its evolution. Multiple detection technologies were operated during the project, including Satellite and Space Debris Laser Ranging, CCD and SPAD photon-counter light curves, as well as measurements from tracking and imaging radars.
CCD photometric measurements are suitable for application of the above-mentioned amplitude method since the relative magnitude is preserved in the light curve, contrary to the case of a photon counter which does not proportionally react to the detected photons. On the other hand, single-photon counter and laser ranging data better fulfil the requirements for the temporal analysis in the epoch method referred above.
The In-Orbit Tumbling Analysis (ιOTA) tool [16][17] was further improved in the current project. The comparison of the propagated attitude with the one determined from measurements serves as validation for the attitude determination methods, the simulator, and the evolution models.

2 Observation campaign

For the observation campaign, different objects have been selected as a representative sample of the tumbling population up to the geostationary altitudes. The list covers:

- Rocket bodies (R/B) and 3 satellites at altitudes up to 1500 km where all optical methods, including the non-cooperative satellite laser ranging, should perform well.
- 2 satellites at altitudes 1500-3000 km.
- R/B and 3 satellites in Medium Earth Orbit (MEO), circular and elliptical.
- 3 R/B in Transfer Orbit and Geostationary Transfer Orbit (GTO).
- 3 satellites in Geostationary Orbit (GEO).
- Special Interest Objects: ERS-2, ADEOS, Envisat.

The orbital configuration of the selected objects assures frequent availability during the terminator period (dusk and dawn) for the photometric observations from Graz and Zimmerwald observatory. It is also required that the closest approach of OOI is frequently above 30° of topocentric elevation to give opportunity for longer passes at higher SNR. The types of targets – rocket bodies and defunct payloads – represent a group of large radar cross-section objects for efficient optical observation and analysis of the tumbling motion. The constraints of the attitude methods require multiple rotations per pass (suggested >10) and some basic information about the R/B technical and optical properties.
In addition to the regular campaign, a joint tracking campaign with the TIRA radar in Wachtberg, Graz SLR station, and the Zimmerwald Observatory was performed. The purpose of the 3-systems tracking campaign was the simultaneous collection of the photometric, laser and radar data.

3 Methods for attitude determination

3.1 Amplitude method

The amplitude method was chosen as one of the methods to be implemented in the current project. Originally the approach was proposed by Williams in [18] and it could later be successfully applied in similar investigations [12]. In the original idea the method applies to cylindrical elongated objects where we assume that a stable attitude state is reached, with rotation around the major axis of inertia, perpendicular to the cylinder axis. The sunlight is diffusely reflected by the lateral surface of the cylinder without considering top and bottom. The existence of maximal and minimal values depending on the observation configuration can be exploited to retrieve information about the orientation of the cylinder axis. The **brightness ratio** between maximal and minimal intensity can be related to a set of possible rotation axes. At least three brightness ratios, obtained observing the same object at different epochs or from different stations, are necessary to reduce the solution set to a unique rotation axis.
Only rocket bodies are a suitable choice for the amplitude method and of the acquired light curves of R/Bs only a smaller subset of objects was appropriate for the intended analysis, since some of them did not show any periodicity and for others there were not enough observations. As an example we show the determination of the rotation axis of the CZ-3B (1998-044B) R/B. One acquired light curve is displayed in Figure 3-1.

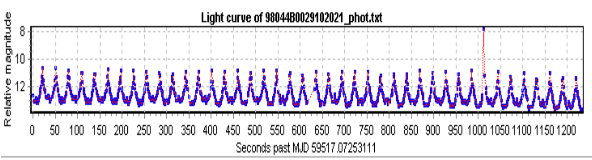

Figure 3-1: Light curve of object 1998-044B acquired on 30.10.2021.

The intensity in the light curves is given in relative magnitudes. Linear detrending was applied to the

analyzed light curves. For the computation of the phase angle TLE elements of the object were propagated to the mid epoch of the light curve.
For the extraction of the brightness ratio, maxima and minima are searched in every periodic interval. Here the period of the light curve is not critical and can be extracted with e.g. a phase dispersion minimization method. Figure 3-2 indicates the result of the procedure to find maxima and minima in the case of the detrended light curve acquired on 30.10.2021. The time and relative magnitude scale have an arbitrary offset, not relevant for the analysis. The extracted period used for the search is 58.45 s. The resulting brightness ratio is -2.38 with a standard deviation of 0.14.
Table 3-1 summarizes the phase angles, extracted brightness ratios and standard deviations for three light curves of this R/B. In Figure 3-3 the set of solutions from the single light curves and the spin axis solution (intersection region) is shown. The latter computation gives the direction coordinates in right ascension RA = 235.2° (converted to positive values) and declination DEC = 64.1°, as well as their covariance $\begin{pmatrix} 24.9 & -3.9 \\ -3.9 & 1.2 \end{pmatrix}$ computed from the statistical distribution of the intersection points.
Although an intersecting solution could be found and it is unlikely that the three sets overlap in the same region by chance, there is no absolute guarantee that the provided result corresponds to reality. Already the assumption of having a constant rotation axis throughout the period of the observations cannot always be ensured. Since in this case the object is in a GTO orbit with a perigee at 625 km, the direction of the rotation axis will be influenced by the predominant gravity gradient torque especially at perigee passage, which during an interval of two months can change the orientation of the axis 30° and more. This error component is mostly present, even for observations within short time, but can be reduced to acceptable levels choosing appropriate time intervals according to the orbit of target.

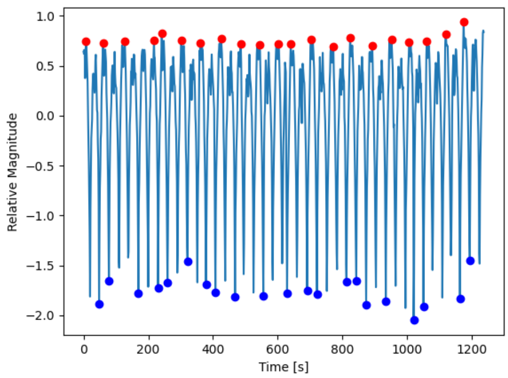

Figure 3-2: Search of maxima and minima of light curve in Figure 3-1.

Epoch	Phase angle [deg]	Brightness ratio	Std. deviation
21-Aug-2021 01:13:00	82	-3.14	0.30
13-Sep-2021 23:10:00	49	-2.81	0.38
30-Oct-2021 01:54:00	32	-2.38	0.14

Table 3-1: Brightness ratios from light curves of object 1998-044B.

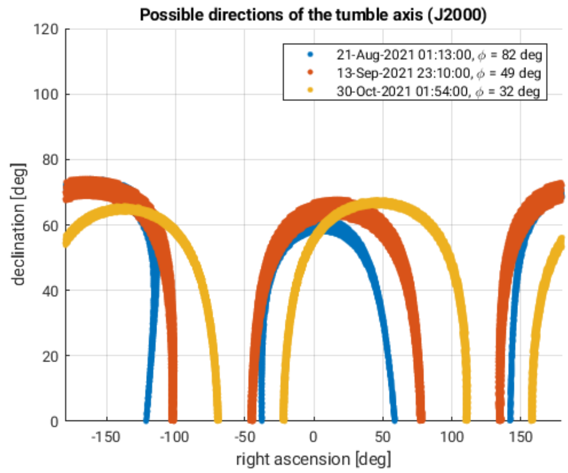

Figure 3-3: Solution sets for the three observations. The region where all sets intersect indicates the spin axis orientation.

3.2 Epoch method

Photometric observations can be collected at different sampling intervals depending on the technology used, from a fraction of millisecond (single-pixel photon counters) to several seconds or minutes (mega-pixel CCD, CMOS imagers). In case of the Graz detection system, the single-photon resolution of the collected optical flux allows recording a high level of signal details of sunlit objects from all orbital regimes with 10 ms binning interval. In addition the combination with high-rate SLR measurements allows for a more complete attitude determination with a lower set of the initial assumptions regarding the satellite shape and an approximate tumbling state. The spinning satellite body causes rotation of the retroreflector array (RRA) resulting in periodic oscillation of the observer – reflector slant range. The spin-related oscillation of SLR data, at the periodicity shorter than the pass duration, can be determined with respect to the best-fit trend function that can be constructed as a low-degree polynomial fit to the SLR data.
The post-processing pipeline for the photometric and SLR timeseries consists of the following steps:

- **Pattern normalization with time derivative**. The Savitzky-Golay filtering (SGF) [19] is used as a method for denoising the raw, observational data.

The SGF is based on local least-squares fitting of the data by low-degree polynomials for smoothing data (improving signal-to-noise ratio) and time derivative calculation.

- **Frequency signal extraction with spectral analysis**. The spectral content of the photometric pattern is extracted by processing the time derivative series with Lomb-Scargle Periodogram [20] which estimates a frequency spectrum of unequally spaced data based on the least squares fit of sinusoids to the data samples.
- **Detrending**. The general timeseries detrending process is based on the Chebyshev polynomials (Gram polynomials) which is a type of discrete orthogonal polynomials used in approximation theory. The degree of the polynomial fit is decided upon the frequency output of the spectral analysis and the pass duration.
- **Inertial spin rate and orientation determination with Phase Dispersion Minimization**. A timeseries folding with Phase Dispersion Minimization (PDM) allows to determine the periodicity spectrum of the observed pattern. In the process, the timeseries samples are folded into 1-degree rotational phase bins and the variation (RMS) of the binned residuals is calculated as a measure of dispersion. Testing range of possible spin period values allows to identify the pattern periodicity by minimizing the dispersion coefficient. The dispersion can be further lowered by correcting the timeseries for the spin vector orientation (removal of the apparent rotation effect).

Several objects were analyzed with the epoch method. We present the example of Jason-2 (2008-032A) with the observation in Figure 3-4. The frequency analysis after the epoch-based light curve post-processing pipeline specified above is shown in Figure 3-5. The apparent rotation angle simulated for the given pass at all inertial orientations of the spin vector is displayed in Figure 3-6 as Mollweide projection. The Theta parameter in Figure 3-7 is a dispersion parameter based on the residuals obtained after the PDM inertial phase folding (apparent rotation effect has been removed). Minimum values of the latter indicate possible locations of the spin axis solution.

The analysis of several objects observed during the campaign showed that, in general, for the spin axis orientation determination a significant apparent motion must be present as a marker of the satellite-observer relative attitude change. In the case of GEO objects, the magnitude of apparent rotation is too small to act as an indication of satellite orientation. This is not a problem for LEOs, where the apparent rotation can exceed 100° per pass thus allowing for the inertial spin axis orientation analysis. However, the accuracy in the latter case turned out not to be enough for cross validation with e.g. the amplitude method.

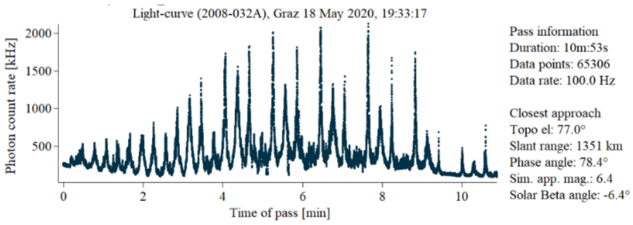

Figure 3-4: Photon counter light curve of Jason-2 acquired on 18.05.2020.

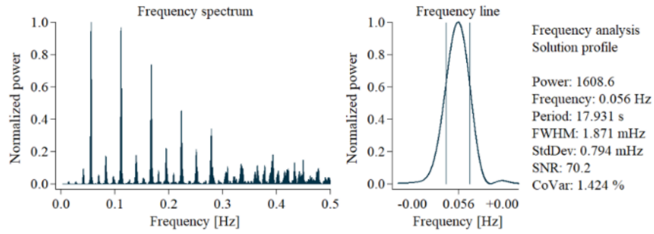

Figure 3-5: Frequency spectrum of light curve in Figure 3-4.

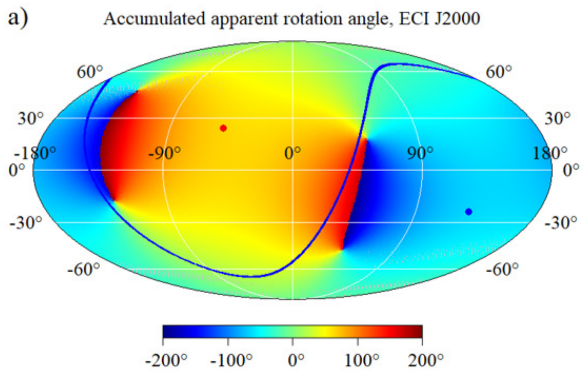

Figure 3-6: Apparent rotation angle simulated over the inertial sphere, ECI J2000. Blue curve is location of the orbital plane; red and blue dots indicate position of the positive and negative normal to the orbital plane.

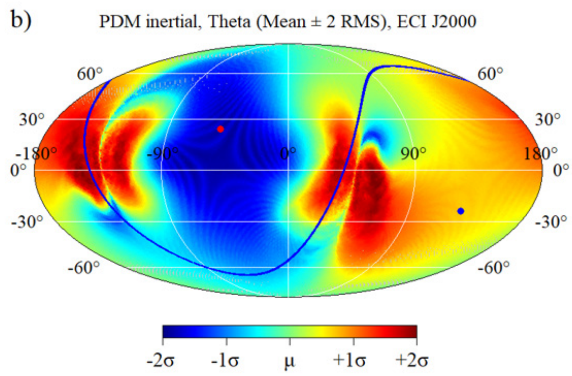

Figure 3-7: Theta parameter of the inertial phase folding expressed in standard deviation units. Minima indicate possible locations of the spin axis.

In addition to the limitations due to the orbital regime, the detrending of light curves is a further challenge in the post-processing as the cause of the trend cannot be predetermined. We have utilized a Savitzky-Golay filter (SGF) that works on a local data sample (rather than full pass) and produces detrended time derivative series that can be post-processed with the epoch methods to determine the satellite tumbling motion parameters. A derivative time series is naturally detrended and distributed about zero level which makes it a perfect input for the epoch method analysis. Moreover, the use of SGF allows for the derivative determination from small data samples, which makes this method suitable for near real-time operation without the need for a full-pass detrending.

4 ιOTA simulations

ιOTA is a highly modular software tool to perform short- (days), medium- (months) and long-term (years) propagation of the orbit and attitude motion (6-DoF) of spacecraft in Earth orbit. Beside improvements in the usability and performance, the models related to Earth radiation and shadow, spacecraft surface reflectivity, eddy currents torque, generic attitude damping, thermosphere wind, and variable solar activity have been extended in the current project. For the computation of the eddy currents torque a magnetic tensor based model has been introduced. The reflection model used for the spacecraft is now material dependent and is considered in both the radiation pressure torque and in the synthetic light curves. Owing to the new modelling capabilities it was possible to validate simulated light curves and different attitude evolution models due to eddy currents, gravity gradient and solar radiation pressure.

4.1 Light curves simulation

We show an example of simulated light curves of Envisat, for which the spin axis orientation was determined during the joint CMOS-SLR-Radar campaign performed on 21.09.2016 [21] and is indicated in Table 4-1. The model of Envisat provided in the ιOTA release was considered with varying reflection coefficients for the different materials listed in Table 4-2. We were able to refine the reflection coefficients to match the dominant peaks in the measured light curves. Figure 4-1 and Figure 4-2 show observed and simulated light curves of the two passes.

Passage	Source	Ang. vel. [deg/s]
2016-09-21 18:53:21	ISAR	0.04, 0.025, -1.659
2016-09-21 20:31:18	SLR	0.0, 0.0, -1.6774

Table 4-1: Assumed solutions for spin axis orientation.

stainless steel	aluminum alloy	polymer composite	radar antenna	solar panel
0.1 / 0.9	0.1 / 0.1	0.9 / 0.1	0.1 / 0.1	0.2 / 0.98

Table 4-2: Diffuse reflection / absorption for different materials of the Envisat model

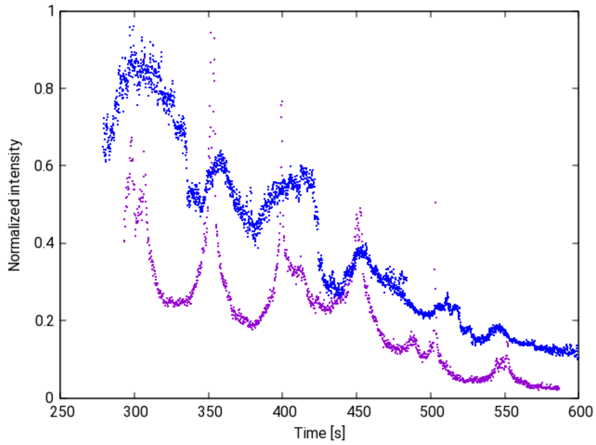

Figure 4-1: Observed (violet) and simulated light curves (blue). Time in seconds since 18:53:21 UTC.

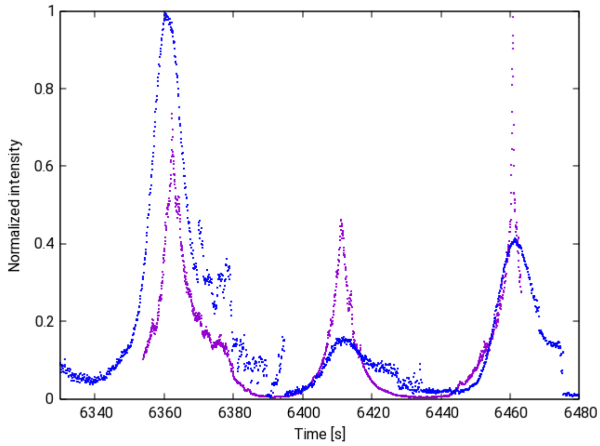

Figure 4-2: Observed (violet) and simulated light curves (blue). Time in seconds since 20:31:18 UTC.

These examples show that to a certain extent it is possible to reproduce the light curve measurements of complex objects with different materials, having an accurate knowledge of their attitude and characteristics. The reflection properties of the materials strongly influence the shape of the light curve and are perhaps the most critical point. These properties are often unknown and even if available from laboratory measurements they might differ from the actual ones in space due e.g. to aging effects.

4.2 Spin rate evolution

The observation campaign provided more than 2'000 measured passes (light curves and SLR data) of more than twenty different objects. Most of the observations were used to determine the spin rate evolution over a period of one to several years. For some objects we checked the consistency of the determined rotation period using measurements from the space observation radar TIRA in addition to the ones from Graz and Zimmerwald. In the case of radar measurements, the period can be extracted from Radar Cross Section (RCS) curves, from Range-Time-Intensity (RTI) plots or through Inverse Synthetic-Aperture Radar (ISAR) images [23].

In Figure 4-3 the rotation period of Jason-2 extracted from observations at the three stations is shown. The period from the radar observation is plotted in red and it seems in-line with the overall trend. While from the radar RCS curves no period could be determined, the range profiles in Figure 4-4 suggest a spin rate around 9 deg/s and the ISAR image (Figure 4-5) confirms this assumption with a spin axis orthogonal to the orbit plane. Note that the ISAR image also indicates that no external satellite damage seems to be present and the solar panels appear perpendicular to each other.

The plot for Jason-2 shows a clear trend with increasing rotation velocity in the last two years. It is supposed that this increase is due to the effect of solar radiation pressure but we were not able to reproduce this trend with simulations.

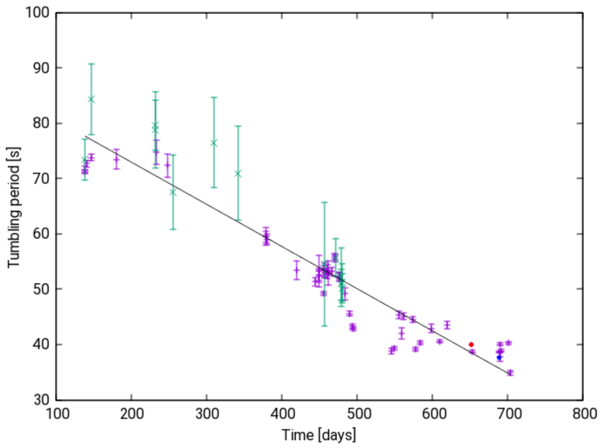

Figure 4-3: Spin period of Jason-2. Time in days from 1.1.2020. Observations from Graz photon counter / SLR (violet / green), Zimmerwald (blue), and TIRA (red).

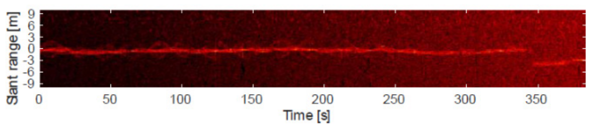

Figure 4-4: RTI plot of Jason-2 indicating angular velocity around 9 deg/s.

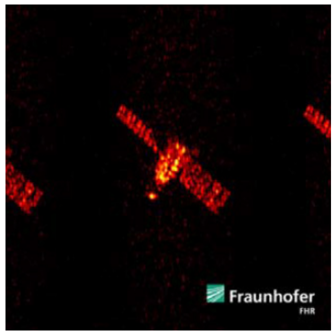

Figure 4-5: ISAR image of Jason-2 assuming spin axis orthogonal to orbit plane and 9 deg/s angular velocity.

Figure 4-6 shows the change of the rotation period of CZ-3B R/B (2019-090D) from combined observations. The period is displayed in logarithmic scale and the fitted line indicates a decay rate of 0.24 yr^{-1}. The line lies within the given error bars and we see that all data consistently follows the same trend.

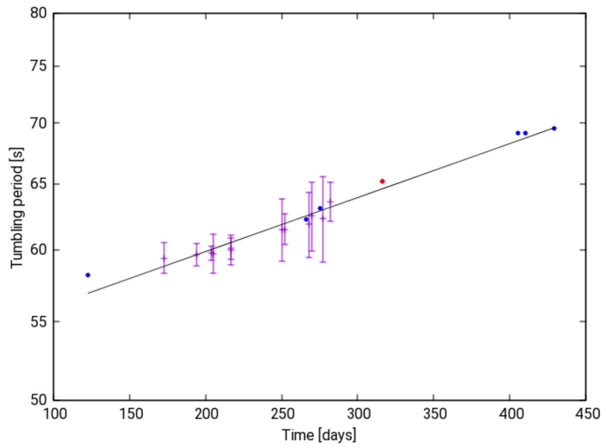

Figure 4-6: Spin period of CZ-3B R/B. Time in days from 1.1.2021. Observations from Graz photon counter (violet), Zimmerwald (blue), and TIRA (red).

The characteristic decay is caused by the damping action of the eddy currents torque. According to [22] we can easily assume that the object is in the decay phase, since usually the flat spin transition phase takes few months, while the stabilization phase show periods of one hour or more, exceeding the length of our measurements. For the simulation with ιOTA we referred to upper stages in a similar category (about 12 m length, 3 m diameter) and we set the maximal inertia to 35’000 kg m^2 and the corresponding component of the magnetic tensor to 4’000’000 S m^4. A simulated decay of about 0.2 yr^{-1} was found, close to the one fitted from the measurements.

5 Conclusions

We have treated different aspects of the attitude motion of space objects and the development of methods to determine their attitude state. Multiple detection technologies were operated including SLR, CCD and photon-counter light curves, as well as radar measurements. Two different methods for attitude determination were considered: amplitude method and epoch method. CCD photometric measurements are more suited to the former method, which is based on the amplitude of the intensity variations in the light curve. Single-photon counter light curves better fulfil the requirements for the temporal analysis in the epoch method, which benefits from the difference between synodic and sidereal rotation rate of the object.

An application of the amplitude method on a CZ-3B R/B was shown. It was possible to find the orientation of the spin axis with an accuracy of few degrees However, the analysis indicates that the formal accuracy does not always correspond to reality if the rotation axis is not constant during subsequent measurements. Ideally, observations separated by short intervals of at most few days should be performed.

Regarding the epoch method, we showed the result of the post-processing pipeline in the case of Jason-2 measurements. In the detrending of the light curves a Savitzky-Golay filter working on local data samples (rather than full passes) and producing detrended time derivative series was successfully used. The apparent rotation angle and the Theta parameter, as a dispersion parameter based on the inertial PDM residuals, were computed. Minimum values of the latter indicate possible locations of the spin axis solution. In general, it turned out that for the spin axis orientation determination the additional requirement of significant apparent motion is the limiting factor. In the cases of GEO objects, the magnitude of apparent rotation is too small to act as a marker of satellite orientation, while for LEOs the apparent rotation should in general allow for a better inertial spin axis orientation analysis.

The new models developed in the ιOTA simulator were used to reproduce light curves and compute attitude propagation. Measured light curves of the Envisat satellite were compared with simulated ones. It was shown that to a certain extent it is possible to reproduce the light curve measurements of complex objects with different materials, having an accurate knowledge of their attitude and characteristics. The reflection properties are the critical point since they are often unknown or not enough accurate.

The spin rate trend of different objects was studied, on one hand to check the consistency of the different type of measurements (Radar, SLR, CCD, Photon counter), and on the other hand, to predict the evolution based on the studied models. An interesting increasing spin rate trend was shown for Jason-2. The spin rate increase is supposed to be caused by the effect of SRP but could not be successfully simulated. Conversely, the evolution of the spin rate of a R/B could easily be explained with the simulated damping effect of the eddy currents showing a typical exponential decay.

Acknowledgements

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