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SHOOTING METHOD TO ALLOW FOR PERTURBATIONS IN THE OPTIMIZED BOUNDARY VALUE INITIAL ORBIT DETERMINATION

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Abstract

Space debris poses threats to functional spacecraft around the Earth due to the possibility of collisions in orbit. The debris object population needs to be cataloged to monitor the space environment. Optical surveys result in observations of objects on very short arcs (when compared to their orbital period). These short-arc angles-only observations are not suitable to derive a reliable orbit, hence two of them are associated together to test if they belong to the same object and to compute initial orbits. Initial Orbit determination (IOD) is done using the Optimized Boundary Value Initial Orbit Determination (OBVIOD), which is an existing method to associate short-arc optical observations. A so-called shooting method is used inside the OBVIOD to include perturbations. This method consists of choosing a hypothetical value for a variable at first boundary and propagating to the second boundary. The propagation from one boundary to the second includes perturbations such as solar radiation pressure, earth's geopotential terms, solar and lunar gravitational forces. The root-finding method used inside the Shooting procedure may take its initial value from the unperturbed solution. However, root-finding methods, like e.g. Newton-Raphson, might have difficulties in the convergence or converge to a wrong solution in case the initial value lies far from the actual root. In addition, for multiple revolutions scenarios several possible solutions, according to the high and low path of the Lambert problem, have to be computed inside the OBVIOD. A root-finding method based on bisection is proposed to get global convergence. Constraints originating from an admissible region approach are set to narrow down the possible scenarios, which are be computed to find the desired solutions. Both, the proposed method and Newton-Raphson are tested for their performance inside the Shooting-OBVIOD. Tests are done using simulated short-arc angles-only observations, separated by single or multiple revolutions, and different area-to-mass ratio values for the observed objects. The results lead to the conclusion that the proposed method is superior to the previous one for use inside OBVIOD to associate short-arc optical observations.
Keywords: Initial Orbit determination, Tracklet association, Shooting method

Nomenclature

α: right ascension
δ: declination
$\dot{\alpha}$: right ascension rate
$\dot{\delta}$: declination rate
ρ_1: range at first epoch
ρ_2: range at second epoch
$\dot{\rho}_1$: range rate at first epoch
$\rho_{2\mathrm{hyp}}$: hypothesis of range at second epoch
$\rho_{2\mathrm{j}}$: range at second epoch
j: iteration number inside the shooting IOD
x_1: value of free parameter in shooting at iteration 1
x_2: value of free parameter in shooting at iteration 2
$\vec{r}_s$: station position
$\vec{v}_s$: station velocity
$\vec{u}$: the line-of-sight vector
$\dot{\vec{u}}$: derivative of the line-of-sight vector
$\vec{v}$: orbital velocity of the object
G: universal gravitational constant
M: mass of the Earth
$\vec{r}$: geocentric position of the object
$\vec{r}_1$: geocentric position at epoch 1
$\vec{r}_2$: geocentric position at epoch 2
$\dot{\vec{r}}_1$: geocentric velocity at epoch 1
$\dot{\vec{r}}_2$: geocentric velocity at epoch 2
a: semi major axis
n: number of individual measurements in a tracklet
m: number of revolutions in an orbit
Ω: right ascension of ascending node
ω: argument of perigee
ν: true anomaly

1. Introduction

Space debris are classified as non-functional, man-made objects in space with no reasonable expectation of assuming or resuming its intended function [1]. It is essential to observe and catalogue them in order to avoid collisions with the active satellites. Optical surveys are conducted to observe the objects in the geostationary region. These surveys yield short sequences of angle measurements, called tracklets, which cover a small fraction of the overall orbit [2]. Siminski et al. [3] proposed an orbit determination method using available information of two tracklets. This approach works with a boundary-value formulation and uses an optimization scheme to find the best fitting orbits (OBVIOD). It solves the Lambert problem, a special case of the orbital boundary value problem, which consists of two position vectors at separate epochs. The IOD in OBVIOD provides an unperturbed solution.

In order to add perturbations in the IOD, a so-called shooting method is proposed here. Following sections will show the working of the latter with two different root-finding procedures and compare their results.

1.1 Working in OBVIOD

The angle measurements consist of series of α and δ values. Linear regression is performed over these series, resulting in average α, δ values and the corresponding $\dot{\alpha}, \dot{\delta}$ for a mean time. This set of values is called the attributable vector [4]. Using the attributable vectors over simple measurements provides an advantage as the angular rates information is now available. Moreover, the mean angular positions, rates obtained from the linear regression will have higher accuracy with respect to the raw observations.

The next step involves a range hypothesis, which is used with the line of sight vectors and station positions to compute position vectors. The Lambert's problem is solved, giving velocities at both the epochs. The angular rates obtained from the previous step are compared with the ones from the attributable vector using a loss function. The latter is based on the difference between the measured and the modelled angular rates scaled by the uncertainty. In this case, the Mahalanobis distance is used as the loss function. For a distribution y, with mean $\bar{y}$ and covariance matrix C_y, the Mahalanobis distance for each point y_i is defined by [5]:

$$D_M(y) = \sqrt{(y_i - \bar{y})^T C_y (y_i - \bar{y})} \qquad (1)$$

A minimization algorithm called Broyden Fletcher Goldfarb Shanno (BFGS) is used to search for the loss function minimum. Press et al. [6] briefly explain the working of this algorithm. If the Mahalanobis distance is below a certain threshold, the tracklets are said to be correlated. In other words, they belong to the same object. The range hypothesis corresponding to the minimum is accepted and the initial orbit is computed for these tracklets. Fig. 1 shows the schematic of OBVIOD.

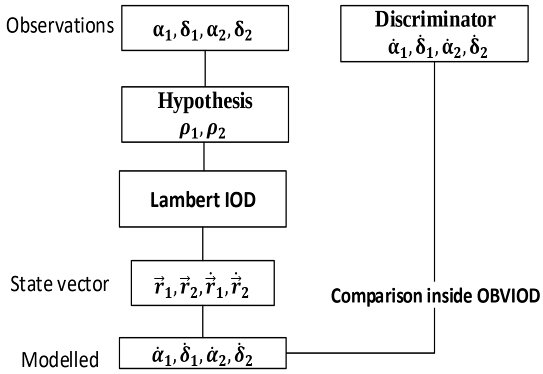

Fig. 1. Process Flow in OBVIOD

2. Shooting method to add perturbations

This method belongs to the class of two-point boundary value problems. It treats the boundary value problem as an initial value problem. It chooses an initial value of the dependent variable at the first boundary, propagates the function to arrive at the other boundary [6]. This solution is compared with the second boundary value. Free parameters at the first boundary are adjusted to satisfy the desired second boundary value. Fig. 2 shows how the different initial values of the dependent variable are taken at the first boundary value in order to reach the desired boundary value.

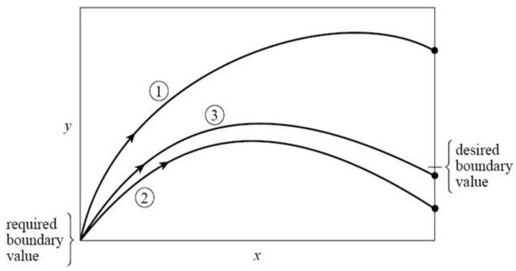

Fig. 2. Schematic of Shooting method

2.1 Shooting method in OBVIOD

The boundary values in the case of Shooting-OBVIOD are angular measurements at both the epochs. Using the attributable vector one has the mean angular positions and rates. The range hypothesis is made for both the boundaries. The station position and velocity at both the epochs is known, the only unknown parameter at the initial epoch is $\dot{\rho}_1$. It is chosen as the free parameter inside Shooting IOD and is hypothesized at the initial epoch. The orbit is computed at this epoch and propagated to the second epoch. The propagation step involves perturbations such as solar radiation pressure, Earth's geopotential terms, solar and lunar gravitational forces. The method is described more in detail in Sections 2.2 and 2.3. The Shooting IOD replaces the Lambert IOD during the minimization of the loss function. The resulting schematic of Shooting-OBVIOD is shown in Fig. 3.

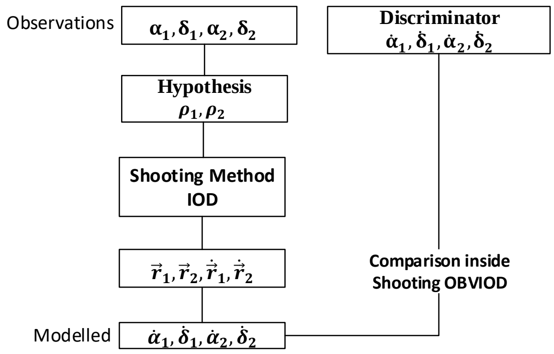

Fig. 3. Process flow in Shooting OBVIOD

2.2 Newton Raphson Method

The Shooting procedure employs a root-finding algorithm to find a solution that satisfies the desired boundary value. Newton-Raphson method is used for this purpose. It extrapolates the local derivative to the next estimate of the root [6]. Hence, the next estimate of the root becomes:

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} \qquad (2)$$

Geometrically it consists of extending the tangent line at a current point x_1 until it crosses zero, then setting the next guess x_2 to the abscissa of that zero-crossing (see Fig. 4).

In case of Shooting IOD, it searches a root for the function ($\rho_{2\text{hyp}} - \rho_2$) using the free parameter $\dot{\rho}_1$. It works well only for local convergence and needs a good starting value. It takes the first hypothesis for $\dot{\rho}_1$ from the unperturbed Lambert IOD so that it does not start too far from the solution. Using α_1, δ_1, $\dot{\alpha}_1$, $\dot{\delta}_1$, ρ_1 and $\dot{\rho}_1$ the orbit is computed, following which the Keplerian propagation is done for half of the iterations. The rest of the iterations take advantage of the propagation with the earlier mentioned perturbations. Once the function value is below tolerance, the iterations are stopped and the corresponding $\dot{\rho}_1$ is accepted for a particular ρ_1 , ρ_2.

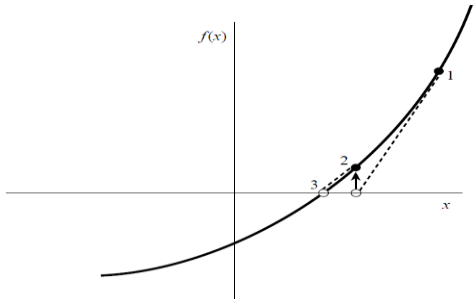

Fig. 4. Function derivative at point x_1 is used to find the next estimate x_2 of the function's root.

This $\dot{\rho}_1$ is used to compute orbit for the ρ_1, ρ_2 inside the BFGS iteration. Fig. 5 shows the scheme of the Shooting IOD using the Newton-Raphson method for the root search.

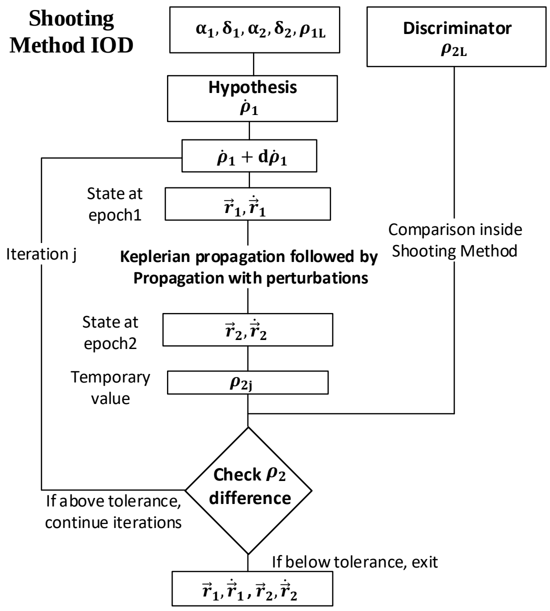

Fig. 5. Flow diagram of Shooting IOD using Newton's method

2.3 Bisection

Newton's method might have convergence issues if the initial estimate is far from the solution. In order to avoid such scenarios, a different root finding algorithm is needed which is more reliable in terms of convergence.

Bisection is one such method, thus it is used to replace the Newton's method inside the Shooting IOD. It works by searching for the point where the function changes its sign. The interval containing the root needs to be identified to begin the search. An interval based on the admissible region for our problem is chosen, see Section 3. A root lies in the interval (a, b) if $f(a)$ and $f(b)$ have opposite signs. The function is evaluated at the interval's midpoint and its sign is examined. The midpoint is used to replace whichever limit has the same sign. After each iteration, the bounds containing the root decrease by a factor of two. If after n iterations, the root is known to be within an interval of size ϵ, then after the next iteration it will be within an interval of size $\epsilon/2$. The iterations are carried out until the function value is below tolerance. Fig. 6 illustrates two points on the function, which constitute the boundary of an interval that contains a root. This interval can also be called a bracket.

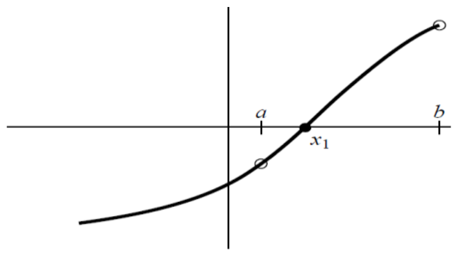

Fig. 6. Bracket (a, b) containing a root x_1 where the bisection method will evaluate function signs at the subsequent midpoints in the iterations until the function value is below a threshold.

2.3.1 Theory and calculation

Brackets are searched in the admissible region defined by the semi major axis, between 41,000 km and 43,000 km. The orbital velocity expression from the vis-viva equation is:

$$v^2 = GM/(\left(\frac{2}{r}\right) - \left(\frac{1}{a}\right)) \quad (3)$$

If the maximum semi major axis value (43,000 km) is substituted in the above quadratic equation, one gets two roots for the velocity. The geocentric position and velocity as a function of ρ, $\dot{\rho}$ can be expressed as:

$$r(\rho) = \vec{r}_s + \rho\vec{u} \quad (4)$$

$$v(\rho, \ \dot{\rho}) = \vec{v}_s + \rho\dot{\vec{u}} + \dot{\rho}\vec{u} \quad (5)$$

The information about r_s and v_s is available. u and $\dot{u}$ are computed from the angular positions, velocities. The only unknown in equation (5), $\dot{\rho}$ can be computed using velocity values one gets from the constraints. Using the values from the semi major axis constraints, one obtains a quadratic in range-rate. Rearranging equation (3) and using $\vec{u}$, $\dot{\vec{u}}$, ρ, $\dot{\rho}$ and $\vec{v}_s$ one gets:

$$a = \frac{rGM}{2GM - r\left(\dot{\rho}\vec{u} + \rho\dot{\vec{u}} + \vec{v}_s\right)^2} \quad (6)$$

The roots of this quadratic will correspond to the bounds of our function by inserting respective a values. For the minimum value only one value of $\dot{\rho}$ is obtained whereas inserting the maximum value results in two values of $\dot{\rho}$. These $\dot{\rho}$ values are used as the starting brackets for the bisection method (see Fig. 7). Point C in the figure corresponds to the semi major axis minimum on the $\dot{\rho}$ axis and points (A, B) correspond to the maximum. Intervals containing multiple roots are separated into smaller brackets. The Lambert's problem has multiple possible solutions for one or more number of revolutions. These solutions are $(2m + 1)$ in number, where m is the number of revolutions. For each number of revolutions higher than zero, there are two solutions. They are called as long-path and short-path orbits. For the same value of semi-major axis, the long-path orbit will have a higher eccentricity than the short-path orbit. G. Zhang et al. [7] give more information about the multiple-revolution Lambert's problem.

In our case, once the brackets containing the roots are found, each bracket is followed one by one starting from the long-path solutions to the short-path solutions. The resulting schematic of the Shooting IOD with the Bisection method is shown in Fig. 8.

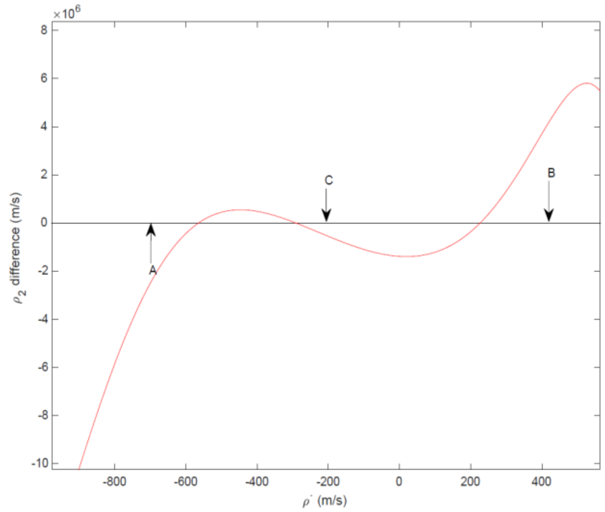

Fig. 7. Brackets for $\dot{\rho}$ resulting from the maximum semi major axis values A, B and the minimum value C.

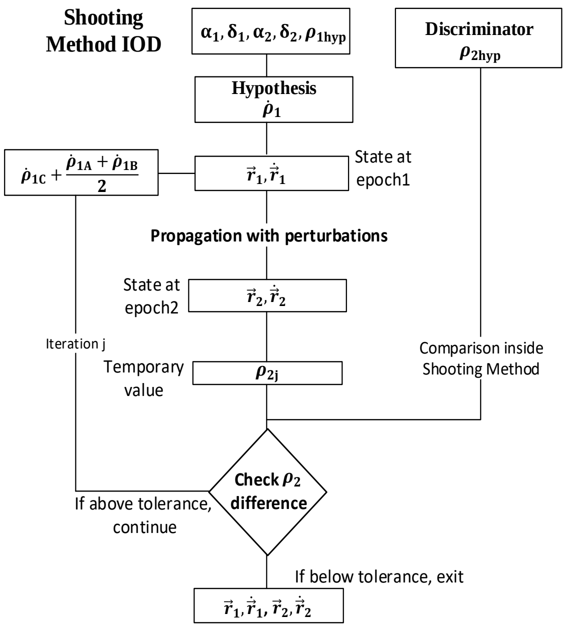

Fig. 8. Flow diagram of Shooting IOD using Bisection method

3. Results and Discussion

3.1 Simulation of Input files

The survey strategy consists of repeatedly scanning a declination stripe with a fixed right ascension. The declination interval is chosen as $\pm 8°$ for each right ascension. The observations are simulated using the coordinates of Zimmerwald observatory, Switzerland to get the topocentric angular positions over a length of 2 to 3 minutes. The two-line-elements (TLEs) used are extracted from the Spacetrack objects catalog [8]. Perturbations are added for geopotential terms, solar radiation pressure, third body attraction forces for Sun and Moon. The Orekit library [9] was used for the force models. In the following tests, observations simulated for the GEO regime are considered. The constraints applied on different orbital elements are: inclination < 10°, eccentricity < 0.3, semi-major axis between 41,000 km and 43,000 km. The Ω, ω and ν are not constrained while simulating the observations. A normally distributed optical error of one arc second was added for all the observations.

3.2 Tests conducted

Tests are conducted by varying the Area-to-Mass Ratio (AMR) of the objects while simulating the observations. The AMR values of 0.01 m^2/kg, 0.1 m^2/kg, and 1.0 m^2/kg are considered. Two tracklets were tested at a time, which were separated by less than one, one or two revolutions. The Mahalanobis distance threshold chosen was 10, based on the strategy mentioned in [4]. The objective is to analyse the correlation performance for both the Newton Raphson method inside the Shooting OBVIOD and the Bisection method inside the Shooting OBVIOD.

Table 1. Results with the Newton Raphson Shooting OBVIOD for low AMR ratio.

No. of revs	AMR	True correlations	False correlations	Missed correlations	Total tracklets
< 1	0.01	11	0	0	22
1	0.01	7	0	4	22
2	0.01	5	0	5	20

Table 2. Results with the Newton Raphson Shooting OBVIOD for AMR 0.1 m^2/kg.

No. of revs	AMR	True correlations	False correlations	Missed correlations	Total tracklets
< 1	0.1	11	0	0	22
1	0.1	7	0	4	22
2	0.1	6	0	4	20

Table 3. Results with the Newton Raphson Shooting OBVIOD for a higher AMR.

No. of revs	AMR	True correlations	False correlations	Missed correlations	Total tracklets
< 1	1.0	11	0	0	22
1	1.0	7	0	4	22
2	1.0	6	0	4	20

Table 4. Results with the Bisection Shooting OBVIOD for the lowest AMR value considered.

No. of revs	AMR	True correlations	False correlations	Missed correlations	Total tracklets
< 1	0.01	11	0	0	22
1	0.01	11	0	0	22
2	0.01	10	0	0	20

Table 5. Results with the Bisection Shooting OBVIOD for the AMR value of 0.1 m^2/kg.

No. of revs	AMR	True correlations	False correlations	Missed correlations	Total tracklets
< 1	0.1	11	0	0	22
1	0.1	11	0	0	22
2	0.1	10	0	0	20

Table 6. Results with the Bisection Shooting OBVIOD for the highest AMR considered in the tests.

No. of revs	AMR	True correlations	False correlations	Missed correlations	Total tracklets
< 1	1.0	11	0	0	22
1	1.0	11	0	0	22
2	1.0	10	0	0	20

Table 7. No. of missed correlations for different AMR, no. of revolutions.

No.of revs	AMR(m^2/kg)	Newton Raphson	Bisection
< 1	0.01	0	0
1	0.01	4	0
2	0.01	5	0
< 1	0.1	0	0
1	0.1	4	0
2	0.1	4	0
< 1	1.0	0	0
1	1.0	3	0
2	1.0	3	0

Table 1, 2 and 3 show the correlation performance when the Newton Raphson method is used for root search inside the Shooting OBVIOD. The true correlations represent the tracklet pairs, which were rightfully associated together. False correlations on the other hand show the cases where two tracklets are wrongly associated even when they do not belong to the same object. The next column of missed correlations shows the no. of pairs that could not be associated together whereas they truly belong to the same object. Table 4, 5 and 6 show the results for the same parameters in case of Bisection Shooting OBVIOD. All the results are briefly summarized in Table 7 to compare the performance of both the methods for all the tested scenarios.

3.3 Significance of the Results

The Shooting method with the Newton Raphson technique is able to correlate all the tracklets, which are only a few hours apart or are separated by less than one revolution. This applies for all the AMR values considered in the tests. However when the tracklets are one or more revolutions apart, it misses some correlations. Since it takes its initial value from the Lambert solution, it tries to find the root closest to the unperturbed solution. As mentioned in Section 2.1, Newton Raphson method works well only for good starting values and may diverge otherwise. For the multiple revolution cases with the increasing no. of possible solutions, this starting value is not accurate enough to lead it to the solution. Moreover, for some high AMR cases the unperturbed Lambert algorithm does not converge and hinders the working of Shooting OBVIOD as well.

The use of Bisection method inside the Shooting OBVIOD makes it less dependent from the unperturbed Lambert solution. In addition, the use of admissible region based on semi-major axis reduces the number of possible scenarios to be computed. For each bracket that contains a root, bisection converges and finds a solution unlike the Newton Raphson method. Furthermore, it is not affected by the initial boundaries chosen for the bracket as long as the latter contains a root. These are the reasons that make the Shooting OBVIOD with the Bisection method for the root search, a better choice for correlation. Furthermore, it is confirmed by the results shown in Table 4, 5 and 6.

4. Conclusions

The aim of this study was to add perturbations in the IOD and choose a suitable root search method, which could be used inside the Shooting method OBVIOD. The working of the Newton Raphson technique inside the OBVIOD was shown and corresponding results discussed. The limitations included inability to converge in single or multiple-revolutions scenarios and sensitivity to the initial value. These issues did not limit the proposed method based on Bisection. The latter is less dependent from the unperturbed Lambert solution and reduces the number of possible scenarios to be computed. This is achieved due to the use of admissible region originating from semi-major axis constraints, which also allows one to focus on a particular orbital regime.

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