Distance between Keplerian orbits in the correlation of short arc radar tracks

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Abstract

Radar observations are used to track space objects in the Low Earth Orbit region. The observations consist of short arc tracks containing range and angular information. The association of two or more observed tracks of the same space object is in general necessary to calculate the orbit of the object with a better accuracy. In the radar case the availability of range and angle measurements allows the computation of an initial orbit from a single track. As a consequence, the association of tracks can be based on a direct comparison of the calculated initial orbits.
In this work a new definition of distance between Keplerian orbits is proposed. The definition extends the existing formulation based on only five Keplerian orbital elements without the anomaly. The modified distance considers an additional term due to the anomaly. The obtained distance can be scaled as a function of the covariance of the orbital elements and can be expressed as Mahalanobis distance. Some application of the distance definition is shown and compared with results obtained using the orbital distance in curvilinear coordinates.

1 Introduction

Validating and improving space debris environment models requires regular monitoring of the space debris population. Today, statistically sufficient data is acquired by conducting optical surveys and by radar beam-park experiments. The data acquisition strategy implies that recent orbital data of the characterised objects are available and are maintained in a catalogue. The build-up of a catalogue and its maintenance depends on the capacity to determine the orbits of the observed objects from few measurements. In fact only a limited number of observations are available per night per object, each over observation arcs that can be as short as a few seconds. Therefore a single track, regardless of measurement type, often does not contain sufficient information in order to reliably estimate the observed object's state or conduct follow-up observations. For this reason the sparse observations or short sequences of observations (tracklets) need to be correlated or associated with each other. Several approaches for optimal association or correlation have been described. The developed methods mainly relate to observations from optical sensors, as e.g. in [RD-1][RD-2][RD-3]. Nowadays several radar sensors are fully or partially devoted to space surveillance. Independent of illumination and weather conditions, radar systems provide observations of space objects in low and medium earth orbits.
In the correlation of radar data Gronchi et al. propose in [RD-4][RD-5] methods to correlate radar data based on the Keplerian integrals, while in [RD-10] the orbits from single tracklets are determined and then compared.
In this paper an association scheme similar to the one proposed in [RD-10] and described in [RD-11] is assumed:

- calculation of initial orbit from radar tracklet
- propagation of orbit to epoch of second tracklet
- comparison of propagated orbit with orbit calculated from second tracklet (orbit matching)
- least squares orbit computation from the observations in the two associated tracklets

To compute the initial orbit the "Range and Angles method" described in the Goddard Trajectory Determination System (GTDS) document [RD-7] is used. The obtained initial orbit has still to be refined with a least squares approach where ranges and angles are weighted differently.
The orbit matching can be evaluated using a definition of distance between two orbits. The matching is then successful if the distance is smaller than a given threshold. Often the definition of Mahalanobis distance is used as a measure of the goodness of the association. The limitation with this measure is in the description of the uncertainty distribution, modeled according to the covariance in a Gaussian distribution. Mostly the Gaussian assumption is enough to describe the uncertainty in the orbital parameters, but depending on the coordinate system the inadequacy can be accentuated (see e.g. [RD-9]). An appropriate coordinate system can be found where the Gaussian assumption approximates the actual distribution. Curvilinear coordinates [RD-8][RD-9] are usually more suitable to describe the orbital uncertainty distribution. Essentially the transformation to curvilinear coordinates takes into account the real curved trajectory of the target.
Other definitions of orbital distance are related to the topological space characterized by the orbital elements. Several studies tried to investigate the topology of the space of Keplerian orbits. Moser [RD-12] first studied the space of constant energy surfaces for bounded Keplerian orbits. He showed that this space is topologically equivalent to the Cartesian product of two spheres. In [RD-13][RD-14] further metrics are proposed as well as applications. An explicit formula for the geodesic distance between points in this space was derived in [RD-15][RD-6].

In this work a new definition of distance is proposed which includes the anomaly in the definition in [RD-6] based on five Keplerian orbital elements. The obtained distance can be scaled as a function of the covariance of the orbital elements and can be expressed as Mahalanobis distance.

2 Distance between Keplerian orbits

In [RD-6] the space of bounded Keplerian orbits of fixed energy is described using the topology V(E) ~ S^2 x S^2, the Cartesian product of two spheres. This topology is extended through the semimajor axis to the cone K(S^2 x S^2). The formula to compute the geodesic distance between points in this space is:

$$d = \sqrt{2(a_1^2 + a_2^2 - 2a_1a_2\cos\psi} \quad (1)$$

where $$\psi = \sqrt{\frac{arccos^2(\vec{\eta}_1 \cdot \vec{\eta}_2) + arccos^2(\vec{\xi}_1 \cdot \vec{\xi}_2)}{2}} \quad (2)$$

and $$\vec{\eta} = \vec{e} + \vec{h}, \qquad \vec{\xi} = \vec{e} - \vec{h}. \quad (3)$$

Here is $\vec{e}$ the eccentricity vector and $\vec{h}$ a normalized angular momentum vector $\vec{h} = \frac{\vec{H}}{\sqrt{\mu a}}$ given the semimajor axis a and the gravitational parameter μ. The related Riemannian metric is induced by the Euclidean metric on $\mathbb{R}^6$. In the article it is mentioned that the geodesic distance on K(S^1 x S^1) can be generalized to a manifold with n spheres K(S^1 x … x S^1) ⊂ $\mathbb{R}^{2n}$. The definition in (2) is then replaced by a general expression which contains not only specifically the angle differences on the sphere for $\vec{\eta}$ and $\vec{\xi}$, but additional angles for any additional sphere:

$$\psi = \sqrt{\frac{\sum_{i=1}^{n} \theta_i^2}{n}}. \quad (4)$$

We want to extend the distance between two orbits (1) to all 6 orbital parameters including the orbit anomaly. We can describe the problem with a topology K(S^2 x S^2 x S^1) ⊂ $\mathbb{R}^6$ where the sphere S^1 is related to the eccentric anomaly, according to the construction in [RD-12]. Then we have in (4) an additional angle difference θ_i. For nearly circular orbits instead of the eccentric anomaly we consider the argument of latitude u, which is more stable w.r.t. correlated errors between true anomaly and argument of perigee.

The formula (1) in $\mathbb{R}^2$ is simply the distance d in the triangle shown in Figure 1 calculated with the law of cosines. If ψ is small d can be approximated by:

$$d \approx \sqrt{(a_2 - a_1)^2 + (a_1\psi)^2} \quad (5)$$

For the generalization in $\mathbb{R}^6$ the eq. (4) for the angle ψ has to be used instead.

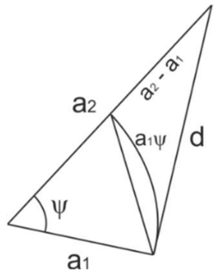

Figure 1: Triangle showing the relation between a_1, a_2, ψ, and d in $\mathbb{R}^2$.

The sum of squares appearing in (4) and (5) suggests the possibility to scale the summands according to their uncertainty as in the Mahalanobis distance. We define the vector:

$$\vec{z} = \begin{pmatrix} a_2 - a_1 \\ \theta_1 \\ \theta_2 \\ u_2 - u_1 \end{pmatrix}, \quad (6)$$

where $\theta_1 = arccos(\vec{\eta}_1 \cdot \vec{\eta}_2)$ and $\theta_2 = arccos(\vec{\xi}_1 \cdot \vec{\xi}_2)$.

To find the covariance matrix $C_{\vec{z}} = Cov(\vec{z})$ we calculate the matrix of the partial derivatives with the components

$$T_{ij} = \frac{\partial \vec{z}_i}{\partial \vec{p}_j}, \quad (7)$$

where $\vec{p} = (a_1, \dots, u_1, a_2, \dots, u_2)$ contains the orbital elements of the two orbits. Setting $C_{\vec{p}} = Cov(\vec{p})$ we have:

$$C_{\vec{z}} = TC_{\vec{p}}T^T. \quad (8)$$

The Mahalanobis distance d_M is obtained from:

$$d_M = \sqrt{\vec{z}^T C_{\vec{z}}^{-1} \vec{z}} \quad (9)$$

3 Simulations

Radar measurements of LEO objects on almost circular orbits (eccentricity < 0.01) at altitudes around 1000 km and 800 km were simulated. The objects from the Space-Track TLE catalogue are observed during one night from a station at 40º latitude. Table 1 shows the values used for the simulation.

Radar pointing	Az. 180º, El. 60º
FoR	Az. 120º, El. 20º
Error (σ) in range	5 m
Error (σ) in angle	15'
Interval betw. obs.	10 s

Table 1. Values for the simulated radar observations for radar tracklets association.

The tracklet association procedure with the above described scheme was applied. In the initial orbit determination only tracklets with at least 3 observations were considered. The Mahalanobis distance in curvilinear coordinate and in the orbit space were calculated and a threshold of 10 was set for both distances. In the least squares calculation of initial and final associated orbit a threshold of 5 for the RMS was chosen as upper limit to discard wrong associations.
After excluding from the total amount of detections the too short tracklets and those where the initial orbit determination fails, a net number of 191 and 769 tracklet pairs remains for the simulations at 1000 km and 800 km, respectively.

4 Results

The correlation performance comparison for different distance definitions was conducted in terms of true and false positives using the simulated data. Table 2 shows the results using the Mahalanobis distance in curvilinear coordinates and in the orbit elements space. For these tests the RMS from the final orbit computation after the association is not used to discard wrong associations. So the ratio true/false positives is only due to the threshold applied to the orbit matching.

	Curv. coord.	Orbit elements
1000 km	179 / 71	186 / 58
800 km	728 / 4136	744 / 4107

Table 2. True and false positives using the Mahalanobis distance in curvilinear coordinates and in the orbit element space.

The results obtained with the orbit elements distance are better than with the distance in curvilinear coordinates. This can also be noticed in the distribution of the Mahalanobis distance in Figure 2 and Figure 3. Using the orbit distance the distribution of true positives is slightly shifted to smaller distance values, compared to the curvilinear coordinates case, while a smaller amount of false alarms is present in the range up to the threshold of 10.

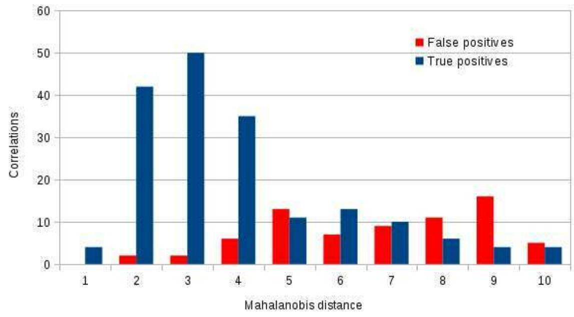

Figure 2: Distribution of distance in curvilinear coordinates for the case at 1000 km.

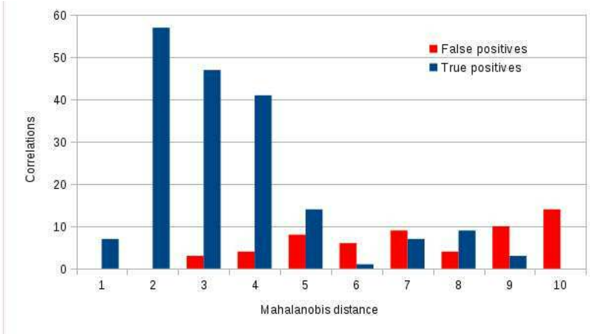

Figure 3: Distribution of orbit distance for the case at 1000 km.

The orbit region at 800 km is much more crowded and this is clearly visible in Figure 4 and Figure 5 where the number of false positives strongly increases at higher distances. The method with the orbit distance still performs better than the one using curvilinear coordinates.

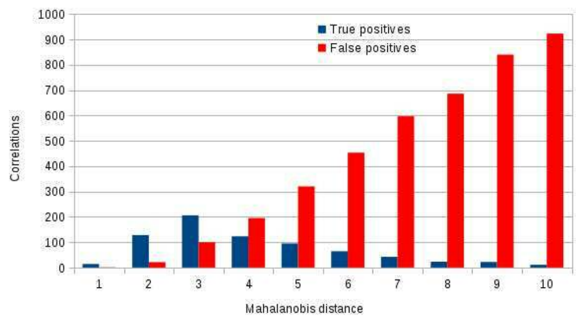

Figure 4: Distribution of distance in curvilinear coordinates for the case at 800 km.

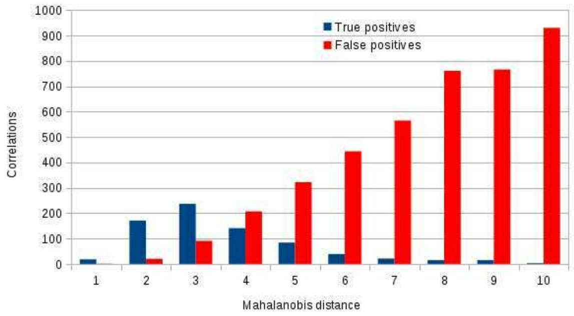

Figure 5: Distribution of orbit distance for the case at 800 km.

The high number of false alarms indicates that a second threshold on the RMS of the final orbit computation after the association is necessary. Table 3 shows the correlation results including the RMS threshold to accept or reject the association. There is a slight decrease in the true positives but a massive reduction of false positives.

	Curv. coord.	Orbit elements
1000 km	156 / 20	159 / 15
800 km	530 / 501	536 / 431

Table 3. True and false positives using the threshold in the distance and in the orbit RMS.

5 Conclusions

Two different distance definitions for orbit matching were compared by assessing the correlation performance. The alternative to the commonly used Mahalanobis distance in Cartesian or curvilinear coordinates is a definition of distance in the space of orbital elements. This space can be described by a different topology and the geodesic distance between two points can be calculated in the defined manifold. The distance between Keplerian orbits can be extended to include the difference in the anomaly. The structure of the equation to compute the orbit distance allows additional weights to be added to obtain a Mahalanobis distance scaled according to the covariance of the orbital elements. The results show that using the alternative distance better true and false positives rates can be achieved. Besides the importance of the distance definition, it is shown that a second criterion for orbit matching based on an RMS threshold is necessary to effectively reduce false associations.

6 Acknowledgements

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