

# Associating optical observations of space debris in GEO with an Elitist Genetic Algorithm

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## Summary

This research aims to provide a method that can treat the association and initial orbit determination problems simultaneously. This problem is also known as the Multiple Target Tracking (MTT) problem. The complexity of the MTT problem is defined by its dimension  $S$ . The  $S \geq 3$  MTT problem is an NP-hard combinatorial optimization problem. In previous work an Elitist Genetic Algorithm (EGA) was proposed as a method to approximately solve this problem. It was shown that the EGA is able to find a good approximate solution in a reasonable computation time. In this work the algorithm is applied to observations taken at the Zimmerwald observatory.

**Keywords:** *tracklet association, initial orbit determination, genetic algorithm, space debris, multiple target tracking*

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## 1 Introduction

Cataloging space debris can be put in the more general framework of Multiple Target Tracking (MTT). The MTT problem can be summarized as follows. A region contains any number of target objects of which the states are unknown. Starting from a set of  $S$  scans, collected by any number of sensors, both the total number of targets and their states have to be estimated. False alarms (sporadic measurements) and missed detections are taken into account. A scan is defined as a set of observations that all originate from different targets. Mathematical formulations of the MTT problem are available.<sup>1</sup>

The problem consists of two interrelated parts, namely data association and state estimation. In the data association part the observations from the different scans have to be associated to the correct targets. The state estimation part then takes these associated groups of observations and estimates the target state. This leads to a search for the permutation that results in the target state estimates that best approximate the measurements, according to a certain metric. The number of scans  $S$  that are used in the problem correspond to its dimension. For a

dimension of  $S \geq 3$  the number of possible permutations greatly increases and the problem becomes NP-hard.<sup>2</sup> For instance, in the case where  $S = 2$  with two observations per scan, there will be a total of seven possible permutations. However for the  $S = 3$  case with two observations per scan, there will be 87 permutations.<sup>3</sup> Several attempts have been made to solve the  $S \geq 3$  MTT problem in an efficient manner. The Multiple Hypothesis Tracking (MHT) algorithm<sup>3,4</sup> seeks to find the optimum solution to the MTT problem by employing a branch and bound methodology. In order for this algorithm to have a realistic computation time the MTT problem has to be simplified. Another approach to the problem is to seek an approximate solution that can be obtained in a realistic computation time. An algorithm has a realistic computation time when it has a Polynomial time complexity. Examples of algorithms that seek an approximate solution are the Lagrangian relaxation technique,<sup>1</sup> and the GRASP algorithm.<sup>5</sup> Another possibility is to use a population based approach. Such an approach aims to statistically represent the debris population. An example of such a method is the AEGISS-FISST method.<sup>6</sup>

The goal of this work is to validate the previously

developed algorithm<sup>7</sup> by applying it to experimental data. A data set of optical observations of the 19.2°E ASTRA cluster is used, all observations are collected by the Zimmerwald observatory.<sup>8</sup>

The paper is organized as follows. First the algorithm is explained in a concise manner. Afterwards, the test case is presented and the results are shown. In the last section the conclusions are drawn.

## 2 Elitist Genetic Algorithm (EGA) applied to MTT

An EGA is a variation on the well known Genetic Algorithm.<sup>9</sup> The difference is that the EGA copies a certain percentage of best solutions from the current generation to the next generation. Any GA needs a fitness function to optimize and an individual to represent a solution in the search space. In this work an individual is represented by a k-matrix as shown in Equation 1.

$$K = \begin{pmatrix} 1 & 0 & \dots & 0 \\ k_{2,1} & k_{2,2} & \dots & 0 \\ \dots & \dots & \ddots & 0 \\ k_{i,1} & k_{i,2} & \dots & k_{i,j} \end{pmatrix} \quad (1)$$

In the k-matrix any entry  $k_{i,j}$  can only have a value of 1 or 0. If  $k_{i,j} = 1$  it signifies that the tracklet in row  $i$  is associated to the object in column  $j$ . The k-matrix is defined in such a way that the first tracklet is always associated to the first object. Following this logic the k-matrix becomes a lower triangular matrix. Besides this, each row may only contain one non-zero element such that  $\sum_{j=1}^N k_{i,j} = 1, i = 1, \dots, N$ . A k-matrix is evaluated according to the fitness function given in Equation 3.

$$f_x = \begin{cases} L_{N \geq 2}(m^*, \bar{p}^*) - \ln \left( \frac{1}{\sqrt{(2\pi)^n |\Sigma_{\Theta}(m^*, \bar{p}^*)|}} \right) & N \geq 2 \\ -(S-N) \ln(1-P_d) - N \ln(P_d) - N \ln(1-P_f) & \\ -\ln \left( (1-P_d)^{S-1} P_d (1-P_f) \right) + \gamma & N = 1 \end{cases} \quad (2)$$

$$f_y = \sum_{x=1}^X f_x \quad (3)$$

Here  $y$  denotes a k-matrix within the current generation,  $x$  denotes a hypothetical object in that k-matrix. The detection and false alarm probabilities are given by  $P_d$  and  $P_f$  respectively. The problem dimension is given by  $S$ ,  $N$  is the number of tracklets used in the IOD. The covariance of the attributed minus computed values is denoted by  $\Sigma_{\Theta}$ . For the  $N=1$  case the tuning parameter  $\gamma$  is introduced. The  $L_{N \geq 2}(m^*, \bar{p}^*)$  is the minimized Mahalanobis distance between the attributed and computed values. The optimized number of revolutions and ranges at the first and last tracklet epoch are given by  $m^*$  and  $\bar{p}^*$  respectively. These values are obtained with the OBVIOD method.<sup>7</sup> The OBVIOD

Table 1: Parameter settings of the EGA (N is the total number of tracklets in data set)

pop. size	$p_{mute}$	$p_{x-over}$	% copied	$\gamma$	$max_{gens}$
2N	1/N	0.5	10	-39	500

method works directly with so-called tracklets. A tracklet is a series of observations spaced closely together (e.g. at 30 second intervals). A line fit is made to the individual observations in the tracklet which estimates the average topocentric angular position and rate. These estimated values are also known as attributed values.

## 3 Results

Satellite clusters are some of the most demanding problems in the field of space debris tracking. The close proximity of the satellites makes it difficult to correctly distinguish them from each other. Consequently it is difficult to determine their state in a correct way. Associating two tracklets at a time can lead to ambiguous results, therefore a  $S \geq 3$  method is beneficial in this case. In Figure 1 the data set can be found.

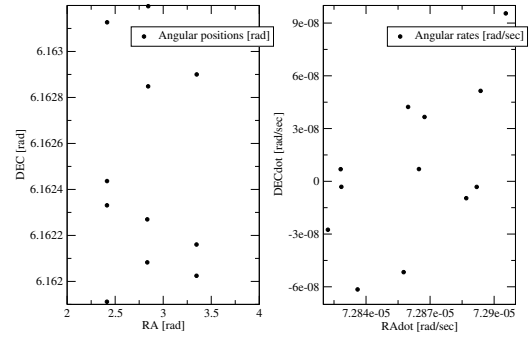


Figure 1: The attributed topocentric angular positions and rates for the ASTRA cluster.

The EGA is applied to the data with the parameter settings listing in Table 1.

The EGA is applied 100 times, the average k-matrix at the end of the run is given in Figure 2. At the end of each run a local search operator is used to further optimize the matrix if possible.

From Figure 2 it can be concluded that the algorithm consistently converges to the same solution. It correctly finds four objects, this coincides with the true number of satellites that are in this specific ASTRA cluster. To validate the results of the OBVIOD method they are compared to those of a least squares estimator. At the AIUB an implementation of the least squares estimator called SATORB<sup>10</sup> is used. Both SATORB and OBVIOD use a Keplerian motion model. The differences in right ascension and declination are found in Figure 3. Figure 4 shows the difference in the angular rates.

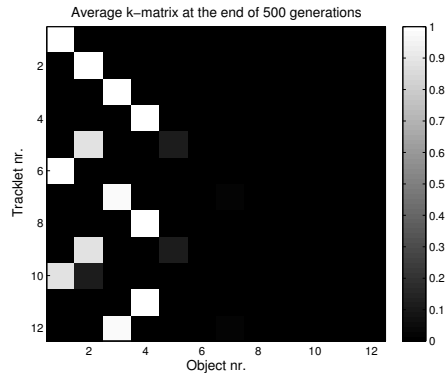


Figure 2: The attributed topocentric angular positions and rates for the ASTRA cluster.

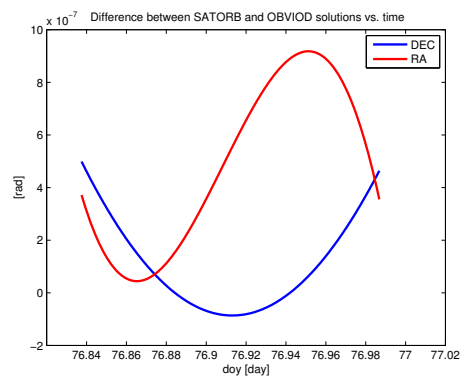


Figure 3: The ephemeris as determined by SATORB and OBVIOD.

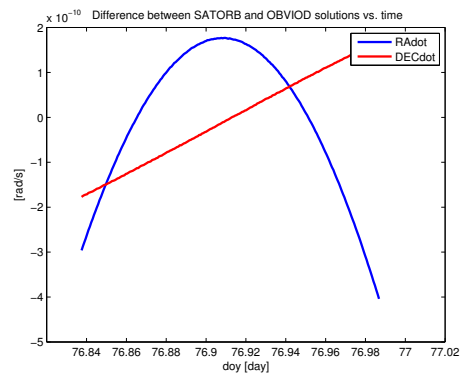


Figure 4: The ephemeris as determined by SATORB and OBVIOD.

#### 4 Conclusions

In conclusion it can be said that the developed algorithm<sup>7</sup> performs as desired on real data. It is able to consistently converge to the same solution and to consistently find the correct number of objects. Also, the OBVIOD method results are consistent with those of a least squares estimator. The differences between these two estimated orbits are

well under the measurement noise of a single optical observation.

#### 5 Acknowledgments

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