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Associating optical measurements of geocentric objects with a Genetic Algorithm: application to experimental data.

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Abstract

Currently several thousands of objects are being tracked in the MEO and GEO regions through optical means. This research aims to provide a method that can treat the association and orbit determination problems simultaneously, and is able to efficiently process large data sets with minimal manual intervention. This problem is also known as the Multiple Target Tracking (MTT) problem. The complexity of the MTT problem is defined by its dimension S. The number S corresponds to the number of fences involved in the problem. Each fence consists of a set of observations where each observation belongs to a different object. The S≥3 MTT problem is an NP-hard combinatorial optimization problem. In previous work an Elitist Genetic Algorithm (EGA) was proposed as a method to approximately solve this problem. It was shown that the EGA is able to consistently find a good approximate solution when applied to simulated data. In this work the algorithm is applied to observations taken by the ZimSMART telescope of the Zimmerwald observatory.

Keywords: Initial Orbit Determination, Multiple Target Tracking, Genetic Algorithm, space debris

Nomenclature

α	Right Ascension
δ	Declination
N	number of tracklets
S	dimension of MTT problem
P_d	detection probability
P_f	false alarm probability
γ	fitness function tuning parameter
K	k-matrix
$k_{i,j}$	entry (i, j) in k-matrix
x	number of hypothetical object in k-matrix
y	number of k-matrix in population
ρ_t	range at epoch t
$\overline{\Theta}$	vector of attributables
m^*	optimized number of orbital revolutions
$\bar{p}^*$	optimized range values (ρ_1, ρ_N)
σ_{ob}	standard deviation of a single observation

Acronyms/Abbreviations

MTT	Multiple Target Tracking
MHT	Multiple Hypothesis Tracking
PBMH	Population Based Meta-Heuristic
EGA	Elitist Genetic Algorithm
GA	Genetic Algorithm
PBIL	Population Based Incremental Learning
DE	Differential Evolution
OBVIOD	Optimized Boundary Value Initial Orbit Determination
IOD	Initial Orbit Determination

1. Introduction

Cataloging space debris can be put in the more general framework of Multiple Target Tracking (MTT). The MTT problem can be summarized as follows. A region contains any number of target objects of which the states are unknown. Starting from a set of S scans, collected by any number of sensors, both the total number of targets and their states have to be estimated. False alarms and missed detections are taken into account. A scan is defined as a set of observations that all originate from different targets. For a mathematical formulation of the MTT problem, the reader is referred to [1]. The fields where MTT problems are encountered are numerous, examples are the tracking of targets in a military context [2], and the tracking of particles resulting from high energy collisions in particle physics [3].

The problem consists of two interrelated parts, namely data association and state estimation. In the data association part the observations from the different scans have to be associated to the correct targets. The state estimation part then takes these associated groups of observations and estimates the target state. This leads to a search for the permutation that result in the target state estimates that best approximate the measurements, according to a certain metric. The number of scans S that are used in the problem correspond to its dimension. For a dimension of S≥3 the number of possible permutations greatly increases and the problem becomes NP-hard [4]. For instance, in the case where S=2 with two observations per scan, there will be a total of seven possible permutations. However for the S=3 case with two observations per scan, there will be 87 of these permutations [5]. A problem can be classified as being either a problem with P or NP complexity. The P stands for Polynomial, which means that it can be solved in a polynomial time. If we say that the problem size is denoted by n then the computation time will be e.g. n^2 (or any other order). The NP stands for Nondeterministic Polynomial time. This means that the computation time is not described by a polynomial but for instance could be 2^n. The computation time of an NP problem will quickly become unrealistically large. Despite its challenges, several attempts have been made to solve the S≥3 MTT problem. The Multiple Hypothesis Tracking (MHT) algorithm [5] seeks to find the optimum solution to the MTT problem by employing a branch and bound methodology. In order for this algorithm to have a realistic computation time the MTT problem has to be greatly simplified. Another approach to the problem is to seek an approximate solution that can be obtained in a realistic (polynomial) computation time. Examples of algorithms that seek an approximate solution are the Lagrangian relaxation technique [1], and the GRASP algorithm [7]. The alternative to solving the S≥3 problem is to solve the S=2 problem. This problem has the favorable computational complexity of O(n^2). Recent work in this area can be found in [8]. These methods evaluate pairs of observations, and take a definitive decision on whether to associate the two observations or not. This can lead to wrong associations, since no information besides those two observations are taken into account (an S≥3 MTT approach would consider more observations, as well as the possibilities of false alarms and missed detections). The severity of this problem depends on the target density in the region of interest. So to solve the MTT problem applied to a densely populated area (e.g. satellite clusters, break-up events) an efficient way has to be found to search through the possible permutations. The MTT problem represents a so-called track based approach, where the state of each object is explicitly resolved. Another possible approach to cataloging space debris is by using a population based approach. Such an approach aims to statistically represent the debris population. An example of such a method is the AEGISS-FISST method [11].

The goal of this paper is to validate the previously developed algorithm [17] by applying it to experimental data. A dataset containing optical observations of the 19.2°E ASTRA cluster is used, all observations are collected by the ZimSMART telescope [18].

Only optical sensors are considered in this work, these provide right ascension and declination values. A few important terms that are used throughout the paper are defined as follows:

- *Observation*: a right ascension and declination pair at epoch t: $(\alpha, \delta)_t$.
- *Tracklet*: a series of seven observations at equally spaced intervals (e.g. 15 seconds), already determined to stem from the same object.
- *Attributed value*: a value that is derived from a tracklet by fitting a line to the individual observations.
- *Attributable*: a set of the four attributed values at epoch *t*: $\left(\alpha, \delta, \dot{\alpha}, \dot{\delta}\right)_t$.
- *Fence*: a set of tracklets that all belong to different objects.

The paper is organized as follows. In the next section the algorithm is briefly outlined. References are given that explain the algorithm in more detail. In section 3 the algorithm is applied and its results are presented. Section 4 consists of the conclusions that can be drawn.

2. Elitist Genetic Algorithm (EGA) applied to MTT with optical measurements

An EGA is a variation on the well known Genetic Algorithm [14]. The difference is that the EGA copies a certain percentage of the best solutions from the current generation to the next generation. Any GA needs a fitness value to optimize and an individual to represent a solution in the search space. In this work an individual is represented by a k-matrix as shown in Equation 1.

$$K = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ k_{(1,1)} & k_{(2,2)} & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ k_{i,1} & k_{i,2} & \cdots & k_{i,j} \end{pmatrix} \tag{1}$$

In the k-matrix any entry $k_{i,j}$ can only have a value of 1 or 0. If $k_{i,j} = 1$ it signifies that the tracklet in row *i* is associated to the object in column *j*. The k-matrix is defined in such a way that the first tracklet is always associated to the first object. Following this logic the k-matrix becomes a lower triangular matrix. Besides this, each row may only contain one non-zero element such that $\sum_{j=1}^{j=N} k_{i,j} = 1, i = 1, \ldots, N$. A k-matrix is evaluated according to the fitness function given in Equation 3.

$$f_x = \begin{cases} L_{N\geq 2}(m^*, \bar{p}^*) - \ln\left(\dfrac{1}{\sqrt{(2\pi)^n \left|\Sigma_{\bar{\Theta}}(m^*, \bar{p}^*)\right|}}\right) \\ \quad -(S-N)\ln(1-P_d) - N\ln(P_d) \\ \quad\quad -N\ln(1-P_f),\ N \geq 2 \\ -\ln\left((1-P_d)^{S-1} P_d \left(1-P_f\right)\right) + \gamma,\ N = 1 \end{cases} \tag{2}$$

$$f_y = \sum_{x=1}^{X} f_x \tag{3}$$

Here y denotes a k-matrix within the current generation, x denotes a hypothetical object in that k-matrix. The detection and false alarm probabilities are given by P_d and P_f respectively. The problem dimension is given by S, N is the number of tracklets used in the IOD. The covariance of the attributed minus computed values is denoted by $\Sigma_{\overline{\Theta}}$. For the N=1 case the tuning parameter γ is introduced. The $L_{N\geq 2}(m^*, \bar{p}^*)$ is the minimized Mahalanobis distance between the attributed and computed values. This function is minimized with the OBVIOD method [17]. The m^* denotes the optimal number of orbital revolutions between the epoch of the first tracklet and that of the last tracklet used in the IOD. The $\bar{p}^*$ is the optimized pair of range values (ρ_1, ρ_N) at epochs t_1 and t_N respectively.

The EGA uses this representation of an individual and the fitness function (3) to evaluate a population of individuals. It uses the classical uniform crossover and mutation operators to produce a new generation. A detailed description of the algorithm can be found in [17].

3. Results

The algorithm is applied to a set of observations that were taken of the ASTRA 19.2°E cluster. This dataset is chosen because it is very similar to the simulated datasets used in [17]. If the results differ from those found in the previous simulated test scenarios, it will be purely because of the difference between simulated and real situations.

Table 1 displays the parameter settings that were used by the algorithm.

Table 1. Parameter settings of the EGA.

Pop. size	p_{mute}	$p_{crossover}$	% copied	γ	Max. gens
2N	1/N	0.5	10	-39	500

All observations stem from the night of 17-18 March 2016 and were made by the ZimSMART telescope. The attributed values are shown in Figure 1.

An EGA is a stochastic method which means that little can be said about its behavior in a deterministic way [14]. Therefore we are forced to study the average behaviour of the algorithm. Figure 2 shows the convergence behaviour of the algorithm. The best fitness value per generation is shown for each individual run as well as the average best fitness value per generation. The plot can be interpreted by considering the average best fitness per generation. The lines that are formed by the individual runs (in gray) give an impression of the uncertainty during the run. This uncertainty is significant. In some cases the optimum solution is found after just a few generations, and in some cases the optimum solution is not found even after 500 generations.

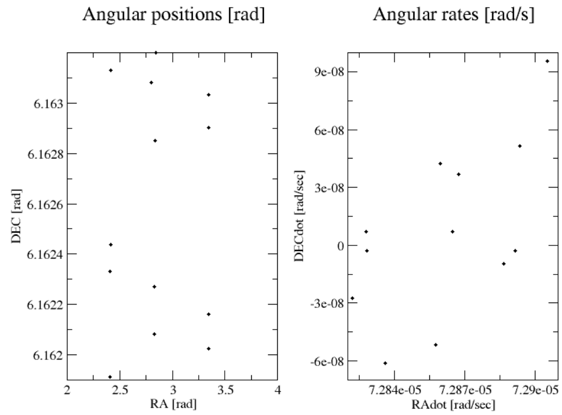

Fig 1. Attributed values of the observed objects in the ASTRA cluster.

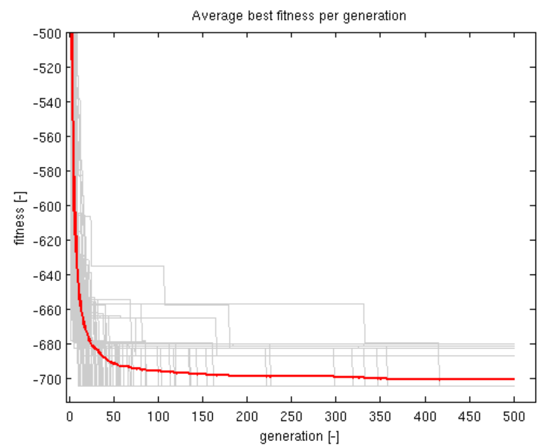

Fig. 2. Average best fitness value per generation, averaged over 100 runs. The results of the individual runs is shown in gray.

The average k-matrix over 100 runs is shown in Figure 3. The result in Figure 3 shows that the EGA works as desired. Even though the uncertainty in the best fitness value per generation might suggest a high uncertainty in the final solution, the average k-matrix in Figure 3 shows that this is not the case. It consistently finds four objects, which corresponds to the number of objects in the ASTRA 19.2°E cluster [19]. The orbit and the RMS of each of the objects is given in Table 2. The RMS is calculated with respect to the individual observations.

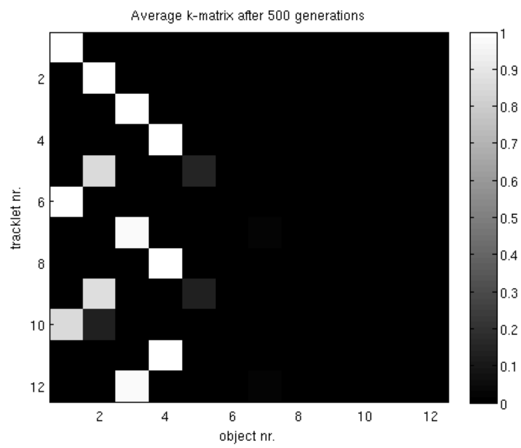

Fig 3. Average k-matrix at the end of 500 generations. The EGA consistently converges to the same solution.

Table 2. Orbit of each object as determined by the OBVIOD method.

object nr:	1	2	3	4
a[m]	4.216e7	4.216e7	4.217e7	4.217e7
e[-]	3.220e-4	5.708e-4	1.967e-4	4.924e-4
i[deg]	0.040	0.044	0.085	0.091
Ω[deg]	103.02	56.622	99.620	66.698
ω[deg]	204.52	-86.36	-48.46	-45.87
M[deg]	-170.56	166.72	85.84	116.20
RMS[rad]	8.86e-7	1.16e-6	1.08e-6	1.14e-6

Table 2 shows that each object is in a near geostationary orbit, which is as expected. The OBVIOD method determines an initial orbit. One purpose of this initial orbit is to provide a good starting point for an orbit improvement. Therefore the difference between the OBVIOD solution and a least squares solution should be small (i.e. on the order of the observation noise) to ensure convergence of the least squares method. Figure 4 shows the difference in right ascension and declination between the OBVIOD and SATORB [20] solutions. SATORB is an in-house implementation of a least squares estimator used for routine orbit improvements at the AIUB. The least squares estimator employs a purely Keplerian model of the satellite dynamics. This is done such that the models used by OBVIOD and SATORB are the same. The uncertainty of a single observation is $\sigma_{ob} = 1''$, which translates to about $5 * 10^{-6}$ [rad]. The difference between the two solutions is smaller than this uncertainty, therefore the initial orbit provided by the OBVIOD method is of sufficient quality.

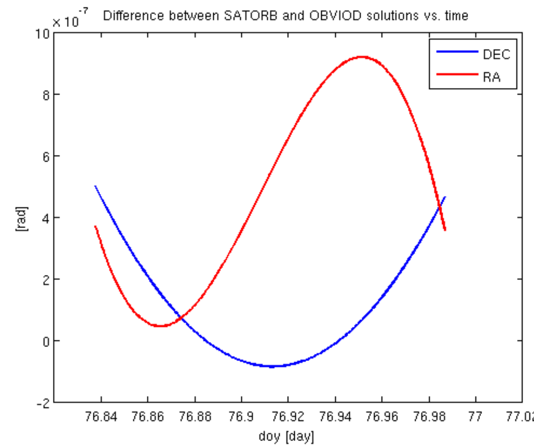

Fig. 4. The difference in right ascension and declination between the least squares solution and the OBVIOD solution.

In Figures 5 and 6 the residuals with respect to the individual observations can be found. They are shown for each of the four objects. An important observation to make is that all the objects have residuals that are consistent to one another. This reinforces the belief that the tracklets are correctly associated to each other.

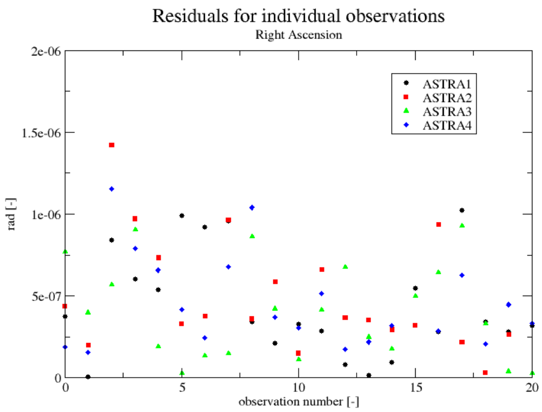

Fig. 5. Residual in right ascension for each observation and for each object.

In Figure 6 an increase is seen in the residuals of the declination for the observations 8-14. This can be explained by understanding the workings of the OBVIOD method. The OBVIOD method uses the first and last tracklets in the orbit determination to define a Lambert problem. Since it uses this Lambert problem the orbit will always precisely intersect the attributed angular positions of the first and last tracklet. Therefore the residuals with respect to the individual observations in the first and last tracklet are also expected to be small. The tracklet that is in between the first and last tracklets is not involved in the definition of the Lambert problem, and therefore the residuals might be larger for those observations.

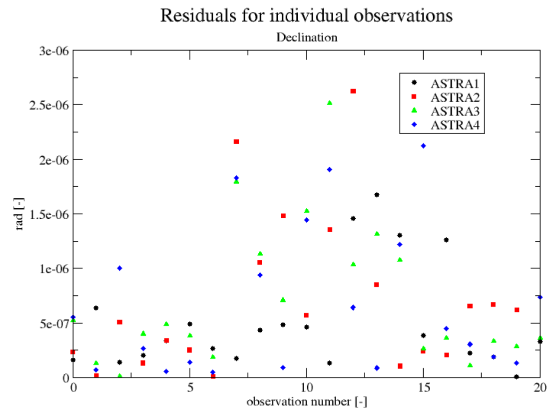

Fig. 6. Residuals in declination for each observation and for each object.

4. Conclusions

The goal of this work is to validate that the algorithm that is developed in [17] works on real data. A dataset containing observations of the ASTRA 19.2°E cluster is used for this purpose. The algorithm is able to consistently converge to the same solution. This solution correctly finds four objects. The orbit of these objects are close to geostationary, which is consistent with the expected results. Each object has comparable residuals which reinforces the belief that the tracklets are correctly associated to each other.

Future work will involve the development and implementation of a search space reduction algorithm. This algorithm will enable the EGA to converge quicker, which allows the EGA to be applied to larger datasets (500-1000 tracklets).

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