Orbit improvement merging angular and laser range measurements

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Summary

The increasing amount of space debris requires huge efforts for the tracking networks to maintain their orbits. The precise knowledge of their positions is fundamental for the planning of collision avoidance maneuvers and future active debris removal missions. The accuracy of an orbit determination process depends on the observables used, their accuracy, the length of the observed arc, and the observer-target geometry. To improve orbits and reduce the needed observation time, the combination of different type of observables is a possible solution.

An in-depth study is carried out to investigate the influence of laser range measurements in the orbit determination process based on the classical astrometric observations. After the validation of the algorithm, the influence of the different observables on the estimated orbital parameters is studied. Then, the effects of the observation geometry and the achievable accuracy in the orbit determination process for high altitude objects are shown. All tests are performed using real measurements provided by the International Laser Ranging Service (ILRS) stations and the Swiss Optical Ground Station and Geodynamics Observatory Zimmerwald owned by the Astronomical Institute of the University of Bern (AIUB).

***Keywords**: space debris, laser measurements, orbit determination accuracy, real measurements.*

1 Introduction

The space operations have a side product: orbital debris. Recent studies[1] proved that the space debris populations will continue to grow, even if we stop to launch new satellites, driven mainly by accidental collisions and breakups. To ensure the long-term sustainability of outer space activities the precise knowledge of the orbit position of space debris is of fundamental importance.

The accuracy of an orbit determination (OD) depends: on the length of the observed arc[2], on the observation geometry[3], on the number and the accuracy of the observations. We decided to study the influence of the high precision Satellite Laser Raging (SLR)[4] measurements in addition to the classical angular ones in the OD process. First, we will show the results of the validation tests of our OD tool modified to handle correctly the SLR measurements. Then, we will report the results obtained by merging real range and angular measurements in different scenarios for different orbital regimes. These studies were performed to identify the influence of the observation geometry, of the length of the observed arc and the effect of different kind of observables on the orbital elements. For these tests only real data provided by the Zimmerwald observatory and by ILRS stations are used.

2 Validation of the orbit determination performed using SLR measurements

The SLR started to be used intensively for the OD in the last 40 years. The two main reasons that make this technique also suitable for the OD of space debris are the fact that no active payloads are needed on board of the satellite and it produces very high accuracy range measurements (~1 mm)[5].

To exploit the SLR measurements we needed to adapt our software used daily for OD. Three main aspects of the tool were improved: the satellite orbit modelling, the corrections to be applied to the measurements and the modification at the normal equation level in terms of partial derivatives and measurements weights. The corrections to the SLR measurements were validated using the Bernese GNSS Software Version 5.2[6]. The solution of our OD process was validated comparing the ephemerides generated by the OD results with only laser measurements with those provided by the ILRS[7]. Table 1 shows the results of the validation tests for Lageos 1. This satellite was chosen because is in a circular orbit at roughly 6000 km of altitude with an inclination of 52°. This kind of orbit altitude provides a good compromise in terms of available number of observations and of size of the visibility window which allows a good coverage of the orbit from the stations. Table 1 shows the length of the arc, the number of observations used in the OD and the mean position differences of 4 days of generated ephemerides (within the OD observation arc) w.r.t. three centers (SGF, JAX and HTS). For completeness, also the mean position differences among the centers are shown. The smallest error, and therefore the best OD results, was obtained using 15 days of observations. In this case, the obtained error and the differences among the centers themselves are comparable. These tests already showed one of the effects of the length of the observation arc: if too short, the error increases due to the small number of the measurements and of stations providing observations (poor geometry conditions); if too long, the error increases due to the weakness of the dynamical model and to the fact that it was not compensated using any empirical acceleration.

Arc-Length	Num. of obs.	Mean Position Difference w.r.t.		
		SGF [m]	JAX [m]	HTS [m]
3 Days	870	0.84	1.47	1.22
7 Days	2044	0.81	1.41	1.15
11 Days	3023	0.72	1.25	0.99
15 Days	4095	0.68	0.94	0.80
19 Days	5034	0.78	1.03	0.93

23 Days	5992	0.74	1.23	0.97
Mean Pos. Difference among centers [m]		SGF Vs HTS	SGF Vs JAX	HTS Vs JAX
		0.47	0.95	0.79

Table 1 Lageos 1 results of validation tests.

Before merging angular and SLR measurements it was necessary to fine tune the weights given to each observables. Due to their different accuracies, we needed to make the system able to take the advantages of both observables without ignoring one or the other. Being, in a LSQ adjustment, the weight of a generic observable $p_i = \sigma_0^2 / \sigma_i^2$. We weighted relatively the two observables putting $\sigma_0 = \sigma_\alpha$, where σ_α is the standard deviation (STD) of the angular measurements (while σ_r is the one for the ranges). Therefore, for the angular measurements the weights is equal to 1 while, for the ranges, the weight is equal to $p_r = \sigma_\alpha^2 / \sigma_r^2$. Both STDs are determined experimentally. For the angles, the mean of the residuals obtained from the system time offset calibration[8] is used. For the ranges, the a posteriori root mean square (RMS) obtained for an optimal OD performed with only SLR measurements is used. While the resulting σ_α is constant to 0.5arcsec for all the tests performed, for the ranges, a particular value was determined for each orbital regime.

3 Orbit determination results

With the next tests we tried to understand the influence on the estimated parameters of the different kind of measurements (SLR and angular), of the relative observer-object geometry and of the number of observations. Finally, we will show the achievable improvements of an OD using 3D measurements w.r.t. that using only angular ones.

3.1. Influence of the number of observations

For space debris objects, due to their shape and their attitude, it is not possible to have the same amount of observations like for a regular geodetic satellite; even if it carries retroreflectors on board. To simulate more realistically this scenario, the following test was carried out with a reduced number of observations. In particular, with this test, we wanted to highlight also the importance of their distribution. Only the measurements (both angular and SLR) acquired during one passage of the Lageos 1 satellite from the Zimmerwald observatory are used. The difference among the different runs is only in the number of ranges (normal points) used (from 0 to 19) and their distribution. The RMS used to calculate the weight of the ranges is 55cm (this RMS is obtained without using any empirical acceleration to remedy to the miss modelling effects). Table 2 summarizes the mean position error and its single components (radial, along and cross-track, namely R, S and W) obtained comparing 12 days of ephemerides. The ephemerides are generated from the OD results performed over one passage of the satellite. The total length of the passage is about 40 minutes, while the ranges are homogeneously distributed over the entire pass; the angular measurements are concentrated in 5 minutes in the middle of it. It is easy to see how already one range produces an improvement of one order of magnitude in the solution. Secondly, the main trend shows how the increase of the number of used ranges produces better results. At the same time, the table shows the importance of the measurements distribution. In fact, 3 ranges (1 at the beginning, 1 in the middle and 1 at the end of the angular series) produce better results than those obtained by using 5 ranges distributed over the entire angular series and 4 times better results than those obtained using all ranges available. This is probably due to the distributions of the ranges and to their higher precision w.r.t. the angular measurements.

Num. of obs.		Mean Error w.r.t. the reference orbit			
1D	2D	R [m]	S [m]	W [m]	Total [m]
-	41	5.925e5	3.263e6	604.1	3.324e6
1	41	995.7	1.461e5	83.55	1.461e5
2	41	7.42	1068	12.23	1068
3	41	6.33	620.2	12.14	620.8
3*	41	5.20	194.9	12.07	193.3
4	41	5.63	356.9	12.10	357.9
5	41	5.26	217.6	12.07	218.9
19*[1]	41	6.02	838.5	6.17	838.7

Table 2 Influence of the number of ranges used over one pass. (* 1 range at the beginning, 1 in the middle and 1 at the end of the angular series,*[1] All ranges availables)

3.2. Influence of the object-observer(s) relative geometry

The following test was performed on the Glonass 123 satellite comparing 16 days of ephemerides and using an RMS for the weight determination of 1.3m. The propagation period is coincident with the OD one. The angular observations were provided by Zimmerwald while the ranges alternatively by Graz, Matera and Mt. Stromlo stations. The first two were chosen since they share the same visibility window with Zimmerwald but are respectively shifted mainly in longitude and in latitude; the third was chosen since it can observe a part of the orbit invisible to Zimmerwald. Table 3 shows the mean errors obtained for the positions and for the osculating orbital elements in the three cases and those obtained using only angular measurements.

10041B		Ang. Only		Zimmer. & Matera		Zimmer. & Graz		Zimmer. & Mt.Stromlo	
Obs 1D-2D		-	63	31	63	33	63	61	63
Position Error [m]	R	43.2		0.382		0.403		1.271	
	S	90.3		13.60		2.165		12.58	
	W	7.10		7.762		6.239		1.294	
	3D	108.5		16.79		6.909		12.90	
Orbital Elements Mean Error	a[m]	0.389		9.404e-2		1.559e-2		7.990e-2	
	e	1.83e-6		1.200e-8		1.500e-8		3.400e-8	
	i [°]	2.38e-5		1.520e-5		1.270e-5		4.300e-6	
	Ω [°]	8.30e-6		2.530e-5		1.990e-5		1.600e-6	
	ω [°]	3.22e-2		3.484e-4		3.414e-4		1.144e-3	
	M [°]	3.22e-2		3.294e-4		3.286e-4		1.171e-3	

Table 3 Geometry influence OD results.

The best solution is the combination of Zimmerwald and Graz with a mean error over 16 days of less than 7m. Looking at each single component, the Graz and Matera cases show similar radial error but different along-track error that is probably due to the gap of 5 days in the Matera data. The Mt. Stromlo case, instead, shows the smallest error in cross-track and consequently in the estimation of i and Ω. Comparing the Graz and the Mt. Stromlo case we can say that: in the first case, the higher density of observation in one part of the orbit helps the estimation of the semi-major axis; while in the second, the observation of opposite sides of the orbit helps in the estimation of the orientation of the orbital plane.

3.3. *Simulation of discovery and follow-up scenario*

The comparison of 6 days of ephemerides in the case of only angular and merged measurements for a GEO object, namely 13034A, is shown in Table 4. For this test, 5 tracklets and 2 ranges over two consecutive observation nights are used. The first 3 tracklets, spread over 3 hours, and the 2 ranges belong to the first night. The angular measurements were provided by Zimmerwald, while the ranges by Herstmonceux. The RMS used for the weight calculation of the ranges is 1.3m. The addition of 2 ranges to the first 3 tracklets produces an improvement of the mean error of 3 orders of magnitude. This ensures the recovery of the object even after 6 days from its discovery. This level of accuracy, as shown by Musci[2], can be achieved with 4 angular follow-ups over 3 observation nights. Adding an angular follow-up in the consecutive night, the improvement is less pronounced but still impressive (from 18.5km to 230m of mean error). Although the ranges are one dimensional measurements and do not provide any direction information, being acquired 2 hours later the last angular observation, they help in the estimation of the orientation of the orbital plane. The angular measurements, which are separated by 1 day, produce an improvement only in the estimation of the semi-major axis.

<table>
<tr><td colspan="2">13034A</td><td colspan="2">1 Night
Ang. only</td><td colspan="2">1 Night
Merged</td><td colspan="2">2 Nights
Ang. only</td><td colspan="2">2 Nights
Merged</td></tr>
<tr><td colspan="2">Obs 1D-2D</td><td>-</td><td>19</td><td>2</td><td>19</td><td>-</td><td>33</td><td>2</td><td>33</td></tr>
<tr><td rowspan="4">Pos. Error [m]</td><td>R</td><td colspan="2">7.193e5</td><td colspan="2">486.8</td><td colspan="2">5.436e3</td><td colspan="2">86.07</td></tr>
<tr><td>S</td><td colspan="2">5.219e6</td><td colspan="2">9.017e3</td><td colspan="2">1.698e4</td><td colspan="2">199.6</td></tr>
<tr><td>W</td><td colspan="2">8.992e3</td><td colspan="2">50.99</td><td colspan="2">703.9</td><td colspan="2">20.87</td></tr>
<tr><td>3D</td><td colspan="2">5.282e6</td><td colspan="2">9.056e3</td><td colspan="2">1.850e4</td><td colspan="2">234.3</td></tr>
<tr><td rowspan="6">Orbital Elements Error</td><td>a[m]</td><td colspan="2">2.793e5</td><td colspan="2">497.2</td><td colspan="2">61.71</td><td colspan="2">6.729</td></tr>
<tr><td>e</td><td colspan="2">4.997e-3</td><td colspan="2">1.822e-6</td><td colspan="2">8.270e-5</td><td colspan="2">3.089e-6</td></tr>
<tr><td>i [°]</td><td colspan="2">4.921e-3</td><td colspan="2">3.190e-5</td><td colspan="2">1.387e-3</td><td colspan="2">4.5e-5</td></tr>
<tr><td>Ω [°]</td><td colspan="2">3.997e-2</td><td colspan="2">2.207e-4</td><td colspan="2">1.407e-3</td><td colspan="2">9.8e-6</td></tr>
<tr><td>ω [°]</td><td colspan="2">22.93</td><td colspan="2">2.995e-1</td><td colspan="2">5.032</td><td colspan="2">1.760e-2</td></tr>
<tr><td>M [°]</td><td colspan="2">16.75</td><td colspan="2">2.865e-1</td><td colspan="2">5.055</td><td colspan="2">1.771e-2</td></tr>
</table>

Table 4 Discovery + follow-ups simulation results.

4 Conclusions

In the space debris environment the precise knowledge of the orbit of an object is of fundamental importance. Since the accuracy of the OD results depends also on the accuracy of the used observations, we investigated the benefits, that the high precision laser ranges could give into the OD process based on the classical angular measurements. First of all, the tool used for the OD at the AIUB was adapted to handle the new observable. After a validation phase and some tuning of the tool, studies were performed to highlight the consequences of the ranges in the OD. First, the influence of the length of the observed arc and the number of ranges used was studied. Then we evaluated the influence of the relative object-observer geometry. Finally, we simulated a classical discovery and follow-up scenario and we highlighted the improvements given by a very small number of ranges. All these tests were performed using exclusively real measurements.

These tests showed the huge improvement achievable using ranges even over a small observation arc. At the same time they highlighted the influence of the relative observer-object geometry and of both observables on each estimated parameter.

This problem needs further investigation, but it has already proved the usefulness of using SLR measurements in the OD for space debris. However, to have more general conclusions we will analyze the results coming from the application to a wider set of observations from different orbital regimes. Further improvement can be given by the investigation of the geometry influence in a more theoretical way. Studies can be carried out using simulation and/or synthetic data, or using a similar approach to that proposed by Cordelli[3] maybe taking into account also the influences of Gaussian mixtures in the OD process.

5 Acknowledgements

Thank to the Swiss National Science Foundation for funding this work (SNF Grant 200020 157062).

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