Association of tracklets from angular and range measurements

Alessandro Vananti*, Thomas Schildknecht
Astronomical Institute (AIUB)
University of Bern, Sidlerstrasse 5, 3012, Bern, CH
Email: alessandro.vananti@aiub.unibe.ch

Jan Siminski, Tim Flohrer
ESA/ESOC Space Debris Office
Robert-Bosch-Strasse 5, 64293, Darmstadt, DE

Beatriz Jilete
ESA/ESAC SSA Programme Office
Villanueva de la Cañada, 28692, Madrid, ES

Summary

A method is proposed to associate series of angular observations, taken with optical sensors, and range/angle observations taken with radars. The association consists in identifying the series of observations, or tracklets, that belong to the same space object. The combination of two tracklets, with angular and range measurements, is an important and sometimes necessary step to determine the orbit of an observed object.

Keywords: *orbit determination, space situational awareness, space debris, tracklet association.*

1 Introduction

In the context of Space Surveillance and Tracking the observations provided by a network of sensors are processed and used to maintain a catalogue of orbital elements. The build-up of a catalogue and its maintenance depend on the capacity to determine the orbit of the observed objects from few measurements sparsely distributed on relative long arcs. The sparse observations or short sequences of observations (tracklets) need to be correlated or associated with each other.
Recent work in this area can be found in the literature[1,2,3]. The developed methods until now are mainly related to observations from optical sensors. Recently some research group has started to investigate the association problem with observations containing ranges and range rates[4].
In this paper a method is proposed to associate radar tracklets (range and angle measurements) with each other and radar tracklets with optical tracklets (angle measurements from an optical sensor, telescope).

2 Proposed scheme

In the typical association problem all the tracklets observed during one or several nights are collected together and all the possible combinations of two tracklets are evaluated according to a certain criterion. The existing methods differ in the way the tracklet is described, e.g. by attributables[1], and in the criterion used to evaluate whether an association is good or not, e.g. based on the Mahalanobis distance between parameters characterizing the two tracklets[2].
The observed tracklet in the radar case can be quite long depending on the Field of Regard (FoR) and the strategy adopted. In the strategy proposed by Mendijur et al.[5], for example, the considered FoR is 20° in elevation and 120° in azimuth. Note that in reality the arc length of the orbit covered by the radar observations is much less than the angles covered in the FoR, especially in the case of low orbits. Nevertheless for relative long tracklets covering several degrees of orbital arc and two or more observations the single tracklet might contain already enough information for a successful initial orbit determination.
If the initial orbit is available for both tracklets, the association of the two orbits can be evaluated comparing e.g. orbital parameters. For the association between a radar and an optical tracklet, only the orbit calculated from the radar measurements should be used. In fact optical tracklets, due to the limited field of view of the sensor and the chosen observation strategy, usually cover a shorter arc and it is difficult with these to compute an initial orbit. If for the optical tracklet a description with attributables is adopted, the orbit of the radar tracklet has then to be compared with the attributable of the second tracklet.
Hence the following basic scheme is proposed:

- Calculation of initial orbit from radar tracklet
- Propagation of orbit to epoch of second tracklet
- Comparison of propagated orbit with optical attributable or orbit calculated from second radar tracklet
- Computation of the associated orbit

In the following these steps are explained in more details.

3 Initial orbit determination

Two methods are considered to compute the initial orbit:

- The Lambert method[7], using two observations with angles and ranges, and the time difference between them.

- The “Range and Angles method” described in the Goddard Trajectory Determination System (GTDS) document[6], able to use more than two observations with an iteration scheme.

The GTDS Range and Angles method provides in most of the tested cases a better initial orbit determination than the Lambert method, probably because it can use all the observations of the tracklet. The obtained initial orbit has still to be refined with a least squares approach where ranges and angles are weighted differently.

4 Propagation

The orbit calculated from the radar tracklet has to be propagated to the epoch of the second tracklet. Also the covariance in the orbital elements, obtained after the least squares procedure for the radar tracklet, is propagated to the epoch of the second tracklet. Here the transition matrix from the first to the second epoch has to be calculated. The covariance is necessary for the comparison with the second tracklet. The propagation is performed in analytical or numerical way:

- Analytical propagation, with better computation performance, but does not consider perturbations.
- Numerical propagation, perturbation are taken into account, but the iterative procedure is in general slower.

In the proposed approach the numerical propagation is used when the inclusion of perturbations is important in the association process, especially if the tracklets are separated by more than one revolution, or the tracked object is at low altitude and is more influenced by gravity perturbations. For this part of the processing the model implemented in the Orekit library[8] is adopted. Earth’s gravity terms up to degree and order 4, air drag, solar radiation pressure, and luni-solar forces are considered. The transition matrix is calculated with a finite difference scheme and is used for the propagation of the covariance.

5 Association of radar orbits

Similarly to the case with optical tracklets, the comparison of radar orbits can be done using the definition of Mahalanobis distance[2] as a measure of the goodness of the association. The limitation with this measure is in the description of the uncertainty distribution, modeled according to the covariance in a Gaussian distribution. Mostly the Gaussian assumption is enough to describe the uncertainty in the orbital parameters, but depending on the coordinate system the inadequacy can be accentuated. For example, in a Cartesian system is difficult to describe the typical “banana” shaped elongation of the error ellipsoid, due to the faster increase in the along-track uncertainty. Several methods have been developed to take into account non-Gaussian distributions in propagation and tracklet association[9,10]. Sometimes an appropriate coordinate system can be found where the Gaussian assumption approximates the actual distribution. The along-track elongation, for example, can be better described in spherical coordinates than in Cartesian coordinates.

The Mahalanobis distance L is given by

$$L = (P_1 - P_2)^T \cdot (C_1 + C_2)^{-1} \cdot (P_1 - P_2)$$

where P_1, P_2 are vectors, in a certain coordinate system, representing the first and second orbit, while C_1, C_2 are their covariance matrices. The use of the above definition for the goodness of an association can be justified in mathematical terms with probabilistic considerations related to the multivariate Gauss distribution[2]. It can be shown that the distribution of Mahalanobis distances, with a Gaussian uncertainty in the vectors, follows a χ^2 distribution with k degrees of freedom, where k depends on the tracklet association method. Based on this property a threshold can be defined below which the association is accepted with a certain confidence level.

In the proposed approach the Mahalanobis distance is calculated in two different coordinate systems:

- Cartesian coordinates, difficult description of the uncertainty distribution
- Curvilinear coordinates[11], more suitable to describe the orbital uncertainty distribution

Curvilinear coordinates are an improvement of the known transformation to the Hill frame, where the position of a moving object (e.g. target), relative to the coordinate system centered at another moving object (e.g. interceptor), is given. Essentially the transformation to curvilinear coordinates takes into account the real curved trajectory of the target. As a consequence in this coordinates system the expected “banana” shaped ellipsoid is not curved any longer and the uncertainty can be better approximated with a Gaussian distribution.

6 Association of radar orbit with optical attributable

In the association of radar and optical tracklets, the propagated radar orbit is transformed into a state vector in spherical coordinates. For the comparison with the optical attributable only the angular spherical subspace is selected, with angular positions and rates, and the Mahalanobis distance in this space is evaluated.

7 Computation of the associated orbit

After the best tracklet association is found, the final orbit using the complete set of observations in the two associated tracklets is computed. Here a least squares improvement of the available radar initial orbit is performed. The root mean square (rms), obtained in the least squares fitting, and weighted according to the average measurement errors, is taken into account to still discard, defining a maximal value, the wrong tracklet associations.

8 Simulations and results

First results were obtained with simulated radar and optical measurements. For the association of radar tracklets a first scenario with LEO objects at around 1000 km altitude with eccentricity smaller than 0.01 is chosen. The objects from the Space-Track TLE catalogue are observed during one night from a station at 40º latitude. Table 1 shows the values used for the simulation. In this scenario the tracklets contain on average 6 observations.

In the association of radar tracklets with optical tracklets a scenario with a GTO population is selected. For the radar observations the same values in Table 1 apply, but a radar pointing at 90º azimuth and 70º elevation from a station at 0º latitude was chosen for better visibility conditions. The settings for the optical observations are indicated in Table 2. With these settings the tracklets consist on average of 30 radar or 20 optical measurements. Furthermore constraints in the observed altitude are set to ensure that the objects are observed at perigee by the radar and at apogee by the optical sensor.

Radar pointing	Az. 180º, El. 60º
FoR	Az. 120º, El. 20º
Error (σ) in range	5 m
Error (σ) in angle	15'
Interval betw. obs.	10 s

Table 1: Values for the simulated radar observations for radar tracklets association

Optical pointing	RA 20º, DEC 0º
FOR (fence)	RA 2º, DEC 30º
Error (σ) in angle	1"
Interval betw. obs.	20 s

Table 2: Values for the simulation of optical observations

The tracklet association procedure with the above described scheme was applied in the two different scenarios. In Table 3 the results are shown in terms of number of simulated associations, number of correct associations (true positives), and false positives found by the procedure. For these results, the initial orbit was calculated with the GTDS method, propagated with a Keplerian model, and for the radar tracklets association, the Mahalanobis distance was computed in curvilinear coordinates. A threshold of 10 in the Mahalanobis distance and a threshold of 5 for the maximal acceptable rms (in the least squares calculation of the final orbit) were chosen.

	LEO	GTO
Associations	280	17
True positives	124	3
False positives	13	0

Table 3: Number of simulated associations and results obtained from the association procedure

In LEO almost half of the simulated associations are found with few false positives. For GTO the number of true associations is lower: the reason probably lies in the difficulty of computing the initial orbits which are in the case of GTO very eccentric.

9 Conclusions

A method to associate radar tracklets on one hand and optical with radar tracklets on the other hand is proposed. The method consists in different steps including: the initial orbit determination from a single radar tracklet, the propagation to the epoch of the second tracklet, and the computation of the Mahalanobis distance to evaluate the goodness of the tracklet association. The very first results show promise and further scenarios will be evaluated with different algorithm options and different threshold values.

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