

Extraction of Spin Periods of Space Debris from Optical Light Curves

Esther Linder, Jiri Silha, Thomas Schildknecht, Monika HagerAIUB, Switzerland, Thomas.Schildknecht@aiub.unibe.ch

The population of space debris increased drastically during the last years. These objects have become a great threat for active satellites. Because the relative velocities between space debris and satellites are high, space debris objects may destroy active satellites through collisions. Furthermore, collisions involving massive objects produce large number of fragments leading to significant growth of the space debris population.

The long term evolution of the debris population is essentially driven by so-called catastrophic collisions. An effective remediation measure in order to stabilize the population in Low Earth Orbit (LEO) is therefore the removal of large, massive space debris. To remove these objects, not only precise orbits, but also more detailed information about their attitude states will be required. One important property of an object targeted for removal is its spin period, spin axis orientation and their change over time. Rotating objects will produce periodic brightness variations with frequencies which are related to the spin periods. Such a brightness variation over time is called a light curve.

Collecting, but also processing light curves is challenging due to several reasons. Light curves may be undersampled, low frequency components due to phase angle and atmospheric extinction changes may be present, and beat frequencies may occur when the rotation period is close to a multiple of the sampling period. Depending on the method which is used to extract the frequencies, also method-specific properties have to be taken into account.

The Astronomical Institute of the University of Bern (AIUB) light curve database will be introduced, which contains more than 1,300 light curves acquired over more than seven years. We will discuss properties and reliability of different time series analysis methods tested and currently used by AIUB for the light curve processing. Extracted frequencies and reconstructed phases for some interesting targets, e.g. GLONASS satellites, for which also SLR data were available for the period confirmation, will be presented. Finally we will present the reconstructed phase and its evolution over time of a High-Area-to-Mass-Ratio (HAMR) object, which AIUB observed for several years.

I. INTRODUCTION

The Astronomical Institute of the University of Bern (AIUB) has wide experience with space debris research, including optical observations of space debris in order to investigate their attitude state through light curves and in order to determine and improve their orbits. The observation facility of the AIUB is located in Zimmerwald, 10 km south of Bern (Switzerland). From the light curves measured in Zimmerwald Observatory, apparent rotational periods and their evolution are estimated for various types of objects (e.g., box-wing spacecraft, upper stages, fragmentation pieces, etc.) in different orbital regions. Currently, there are several different instruments available at the Zimmerwald Observatory to acquire photometry of space debris objects, and there are several different established processing techniques used to extract apparent spin periods from light curves. These instruments, as well as the light curve processing techniques will be further discussed.

Light curves are time series which usually contain information about the change of magnitude (or intensity) over time. Light curves are commonly used in the astronomical community to determine physical characteristics of minor planets, namely their rotation rate, spin axis direction, or shape. Additionally they can be used to detect the presence of minor planets moons, e.g. binary minor planets. In space debris domain the

light curves are the major source of information for the attitude state of non-controlled objects. They are used to determine whether the objects are stable or tumbling. In case of tumbling behaviour, the apparent spin rates can be extracted. Light curves can be acquired for any type of object which is illuminated by the sun and is reflecting enough light to be visible from the ground. Light curves, together with the technique of phase reconstruction, can be used for object correlation. Under specific conditions light curves can also be used to determine the spin axis direction¹. Different types of measurements, e.g., Satellite Laser Ranging (SLR) ranging (to cooperative targets)², light curves and information extracted from them can serve as a mutual cross-check or can complement each other in order to precisely determine the attitude state of the satellite. Last but not least, light curves can be cross-check with forward attitude modelling. Synthetic light curves can be generated and then compared with real observations for final model validation³.

II. ZIMMERWALD OBSERVATORY

AIUB's Zimmerwald Observatory consists of three main optical systems, the 1-m Zimmerwald Laser and Astrometry Telescope (ZIMLAT) (Fig. 1), the 0.2-m Zimmerwald Small Aperture Robotic Telescope (ZimSMART), and the 0.5-m experimental ZimSPACE telescope.

ZIMLAT is used either for SLR to cooperative targets or for optical observation (astrometric positions and magnitudes) of artificial and natural objects in near-Earth space. During daytime the system operates in SLR mode only. During night time the available observation time is shared between SLR and CCD/sCMOS⁴ based on target priorities. The switching between the modes is done under computer control and needs less than half a minute.



Fig. 1: AIUB's 1-meter ZIMLAT telescope dedicated to the photometric and astrometric measurements.

Routine photometry measurements are currently mainly performed with ZIMLAT, but occasionally experimental photometric observations using a so-called "streak" approach are also performed. For this purpose the ZimSMART telescope is used because of its wide field of view (FoV)⁴.

III. APPARENT SPIN EXTRACTION METHODS

Till September 2015 AIUB/Zimmerwald acquired more than 1,300 light curves with ZIMLAT of objects placed on Low Earth Orbit (LEO) to Geosynchronous Earth Orbits (GEO). To be able to extract reliable information from these light curves, namely apparent spin rates, new software had to be developed.

III.I Review of methods

The first three implemented methods for time series analysis were the Fast Fourier Transformation (hereafter FFT), the Periodogram analysis and Welch's method. These three approaches to detect periods from light curves were chosen as a starting point to get familiar with time series analyses which are widely used in scientific community. All three mentioned methods are based on the Fourier transform and need therefore equally spaced data in time as an input. For that reason light curves need some pre-processing to be further processed. However, there are several types of approaches which are able to deal with unevenly spaced

data. One group is so-called Folding methods like Epoch folding or the Lafler-Kinman method. After further investigation it was decided to implement the Epoch folding, because the results shown by S. Larsson⁵ preferred this method over Lafler-Kinman method. Another approach is the Lomb-Scargle periodogram. This analysis showed its strong reliability for period estimation from light curves⁶ and from SLR residuals measurements². Additional promising method was investigated, namely the D-approach⁷, but unfortunately deeper mathematical background for this method could not be retrieved.

Additional methods were further reviewed. The continued wavelet analysis is a tool to detect non-stationary processes⁸. Since our task was to extract a periodical signal from time series, it was decided that the continued wavelet analysis should not be implemented. Singular spectrum analysis has been developed in geophysics to deal with short (a few dozens data points) and noisy time series⁹. However, AIUB's light curves data are usually not very noisy and if necessary, one can increase the number of data points per light curve by extending the observation campaign. For that reason for now we did not implement the singular spectrum analysis into our software. D. A. Leahy et al.¹⁰ found that the Rayleigh test is an appropriate method to search for the broad pulses which are typical for accreting, pulsing X-ray sources. But light curves are the time series which is of different shape. That is why it was decided not to implement the Rayleigh test. The Pattern analysis discussed by C. Früh and T. Schildknecht¹¹ is not able to recover a pattern which is not complete. This can happen if a gap is present in the data set, which often the case for AIUB's light curves and for now it was decided not to use given method.

One of the final products of apparent spin period analysis is the light curve reconstructed phase, which is also sometimes referred to as phase diagram or phased plot. The shape of reconstructed phase is directly related to the geometry and surface properties of the target. Finding the phase of the object and finding the object's apparent spin rate are strongly related problems. Once the apparent spin rate is determined, it can be used to construct the phase. However, also inverted approach can be applied, where by finding the phase, the apparent spin period can be determined as a side product. In case this method is automatized it can be used as a single light curve analysis method.

Finally, there were six methods in total implemented in our program for light curve processing analysis. These methods were the FFT, Periodogram analysis, Welch's method, Epoch Folding, the Lomb-Scargle periodogram and the phase reconstruction method.

III.II Testing of methods

To learn the robustness but also limitations of each method, they were tested on generated type of data like sinusoidal functions and synthetic light curves. The sinusoids were generated by using sinusoidal function or combination of sinusoidal functions for which we knew the input periods. Such a time series was then analysed with different type of methods in order to investigate method's reliability. Example of a generated sinusoidal time series is plotted in Fig. 2. This time series was generated by combining two sinusoidal functions with periods equal to 110.3 s and to 190.6 s. For tests purposes added was also simulated measurement noise following the normal distribution.

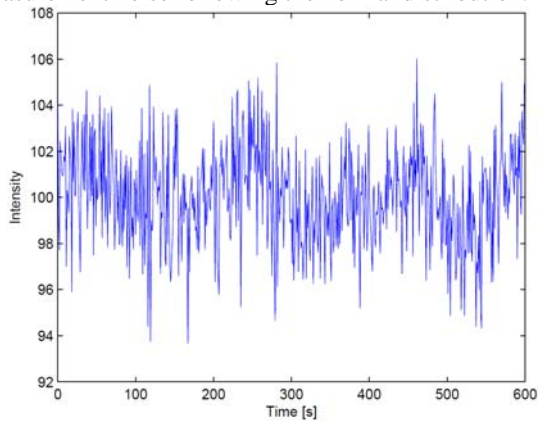


Fig. 2: Example of a simulated light curve by combining two sinusoid functions and by adding simulated measurement noise.

The synthetic light curves were generated by using the 3D render called Blender 2.69¹² and by AIUB created 3D models of a cylinder and a cylindrically shaped upper stage, and two plate-like fragments pieces with different surface reflectivity properties. These light curves had equally and unequally spaced intensity measurements, with and also without measurements gaps, which are usually present in the real data sets due to the technical reasons. Several tumbling scenarios were simulated for each of the 3D model, and for some cases also different phase angles (angle between the Sun, target and observer) were assumed. The 3D models which we used to generate the synthetic light curves are plotted in Fig 3.

For majority of the cases it was assumed a precession of the rotation axis, so two different apparent spin rates were presented in the synthetic light curves. In Fig. 4 is plotted an example of synthetic light curve for the upper stage model, the complete series (upper figure), and series with randomly generated gaps (lower figures). For this specific scenario assumed was rotation period equals to 39.3 s and precession of spin axis equals to 139.1 s.

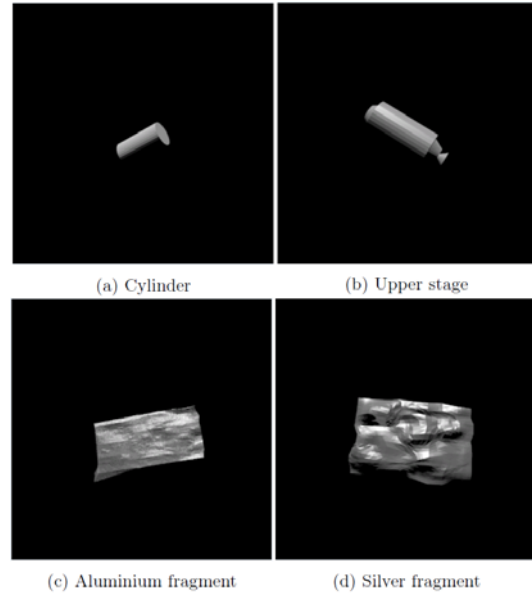


Fig. 3: 3D objects used to generate synthetic light curves for investigation of time series analysis methods reliability.

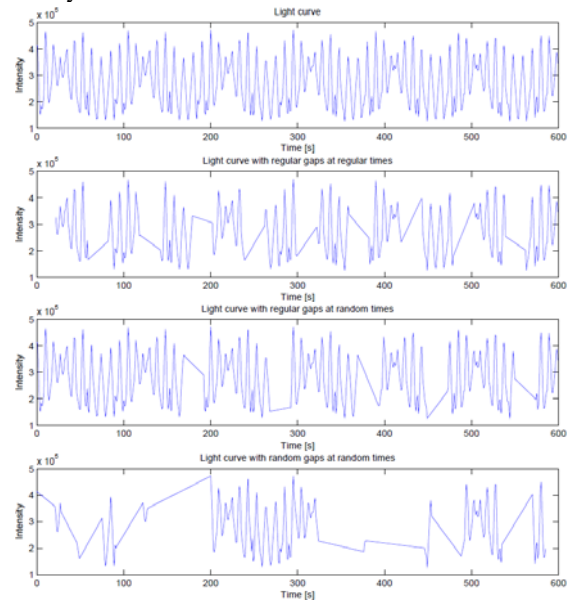


Fig. 4: An example of a synthetic light curve generated for cylindrically shaped upper stage.

Fourier based methods

Fourier based methods were tested on sinusoidal functions, as well on synthetic light curves. An analysis using one and two combined sinusoids was done, where the period with the highest power in power spectra was compared to the known input period(s). Investigated were also the frequency resolutions given by the sampling frequency and the number of data points.

The FFT, Periodogram analysis and Welch's method were found to reveal half of the spin period as first valid period in their power spectra for a period length up to 11 % of the whole time series length. This was found for evenly and unevenly spaced intensity measurements, where the unevenly spaced intensity measurements were interpolated. Welch's method showed that its behaviour is most steadily and the valid period in the power spectra (period with highest peak) was independent of object's 3D shape, its reflection properties, or even phase angle. As soon as there were gaps present in the data, the detection of the input known period was hard or even impossible for this method.

The performed tests revealed that the Fourier based methods capability to determine the period depends on the ratio between the time series length and the period size. As far as the time series length is at least six times longer than the period itself (or period is shorter than ~16.7 % of the whole time series), all three methods are able to reliably determine the period.

For real light curves analysis with Fourier based methods it is recommended to use interpolation to make the data equally spaced, which is a requirement for all the Fourier based methods. If the apparent spin period is a priori known before the observation is acquired, e.g. first look on the object was already performed with streak photometry approach^{1,4} or object was already observed in the past, it is recommended to acquire at least six times longer time series than is the expected length of the apparent spin period. This should make the Fourier based methods results more reliable.

Epoch folding method

The Epoch folding was tested with sinusoids and with simulated light curves. The detection of a shorter period was not influenced by the presence of a larger (precession) period in the light curve. To detect a period or half of its value was possible up to a period length of 45% of the whole light curve, which depends on the quality of the data set (see Fig. 4 for different cases). This method needs an input period to start with. This can be either set manually by the user after the visual investigation of the light curve or values determined by other methods can be used as input, e.g., from Fourier based-methods. In that case the Epoch folding can serve for a refinement of results found by other methods.

Lomb-Scargle method

Sinusoids and simulated light curves were used to test the Lomb-Scargle algorithm. For periods up to 40% of the length of the light curve, the period or half of its value was detected with the Lomb-Scargle algorithm for light curves without gaps (example is plotted in Fig. 4, first upper figure). For light curves with gaps (examples plotted in Fig. 4, three lower figures), periods were detected up to 16% of the whole time series length,

independent of the shape of the light curve. In some cases more than one solution was found, and a cross-check with other methods was necessary.

Phase reconstruction method

The phase reconstruction was tested only on the real light curves. Once the apparent spin period is determined with any of the previously mentioned methods, it has to be confirmed by the phase reconstruction. Currently, this is done in a semi-automated way where some user interaction is still necessary.

Pre-processing methods

Sometimes the light curves have to be pre-processed. This has to be done especially for Fourier based methods. In our case we used a simple linear interpolation between measurement points in order to get in time equally spaced intensity data.

For some objects, mostly for LEO, the aspect angle changes rapidly. This causes a strong trend in the light curve. These trends can be partially removed by applying detrending methods based either on a physical approach or mathematical approach. AIUB is currently using one kind of both. Either there is assumed a Lambertian sphere and current position of the Sun, object and observer. This can in some cases partially or fully remove the trend. Other option is to fit the data by polynomial function. Then, by subtracting the fitted polynomial the trend can be partially cancelled.

Methods summary

In Table 1 are summarized major properties, advantages and limitations which were identified for all previously discussed methods.

IV. EVALUATION OF REAL LIGHT CURVES

The real light curves analysed for this work were collected with the ZIMLAT telescope. During the evaluation of light curves, we used Fourier based methods, Lomb-Scargle analysis and Epoch folding to provide an input spin period for the phase reconstruction. Iteratively, we then looked for the best reconstructed phase to confirm and/or improve determined apparent spin period.

We investigated different types of objects like spacecraft, upper stages and debris pieces, with orbits from LEO to GEO.

GLONASS objects

GLONASS satellites are box-wing type of spacecraft with cylindrically shaped bus and two solar panels attached to it. For box-wing type of spacecraft there have been many times observed tumbling type of behaviour, after the satellite become non-active⁶.

FFT, Periodogram, Welch's method	
Period detection	Able to reliably extract apparent spin period if the period is less than 16.7% of the whole time series length for light curves without gaps.
Advantage	Can be used as a cross-check with e.g. Lomb-Scargle periodogram.
Disadvantage	Limited period resolution, needs equally spaced data, problems processing light curves with gaps.
Epoch folding method	
Period detection	Able to reliably extract apparent spin period if the period is less than 45% of the whole time series length
Advantage	Does not need equally spaced data.
Disadvantage	In some cases not very reliable, sometimes necessary to input initial value, e.g. determined from other methods.
Lomb-Scargle periodogram	
Period detection	Able to extract apparent spin period if the period is up to 40% of the whole time series length for light curves without gaps and less than 16% of the whole time series length for light curves with gaps..
Advantage	Very reliable results, applicable also for light curves with gaps and irregular measurements. Does not need equally spaced data.
Disadvantage	Results not always conclusive, sometimes necessary cross-check with other methods.
Epoch reconstruction method	
Period detection	Sometimes able to extract apparent spin period if the period is less than the whole time series length (less than 100 %).
Advantage	Very reliable results, applicable also for light curves with gaps and irregular measurements. Does not need equally spaced data.
Disadvantage	Not applicable for noisy data. Needs input from other methods as a starting point. Manual interaction necessary in many cases, but the whole process can be automatized. Less efficient when there is trend present in data.

Table 1: Summary of investigated methods properties, their advantages and disadvantages.

To May 2015 there were identified by the CODE (Center for Orbit Determination in Europe) centre¹³ 70 GLONASS satellites from GLONASS global navigation system which are already decommissioned. For that reason during the spring and summer 2015 AIUB performed an observation campaign with ZIMLAT telescope in order to obtain light curves of these defunct GLONASS satellites, extract their apparent spin rates and construct their phases.

Finally, for 66 satellites there was a least one light curve available with length in average of 10 min and except few cases the tumbling was observed for every of withdrawn GLONASS satellites. For nine of them the spin rate was below 60 s. The fastest observed spinning GLONASS satellite was G41 (1989-001B) with apparent spin rate equals to 9.26 s, which corresponds to angular velocity equals to 38.9 deg/s. This value was determined for the date 2014-12-22. This satellite's light curve as well its reconstructed phase for different investigated period values can be seen in Fig. 5. For completeness, the GLONASS for which stable or slow spinning (apparent spin rate > ~10 min) behaviour was observed were usually decommissioned within the last six years.

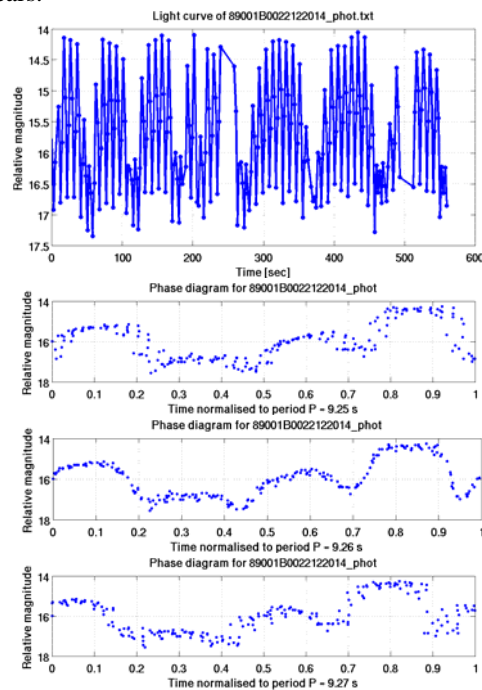


Fig. 5: Light curve (upper figure) and its reconstructed phase (lower figures) by using input period values 9.25 s, 9.26 s and 9.27 s for object 89001B which was observed on 2014-12-22 by ZIMLAT.

We applied discussed methods to determine all the GLONASS satellites apparent spin rates and to reconstruct their phases. In Fig. 6 are plotted reconstructed phases for GLONASS 62 (1994-021A), G63 (1994-021B) and G64 (1994-021C). These satellites were launched together in 1994 and their shape/geometry is expected to be the same. According to CODE centre data the satellite 94021A was withdrawn at 2000-09-30, 94021B in 2002-12-24 and 94021C in 2000-01-15. Extracted apparent spin periods were 118.85 s (94021A), 42.8 s (94021B) and 109.51 s (94021C). Reconstructed phases of all tumbling

GLONASS satellites showed a structure where always four peaks are present. This is also the case for phases showed in Fig. 6. However, the shape is slightly different from case to case. This is caused by the different aspect angle which is affected by the current attitude state of the spacecraft, current position of the observer and position of the Sun illuminating the target.

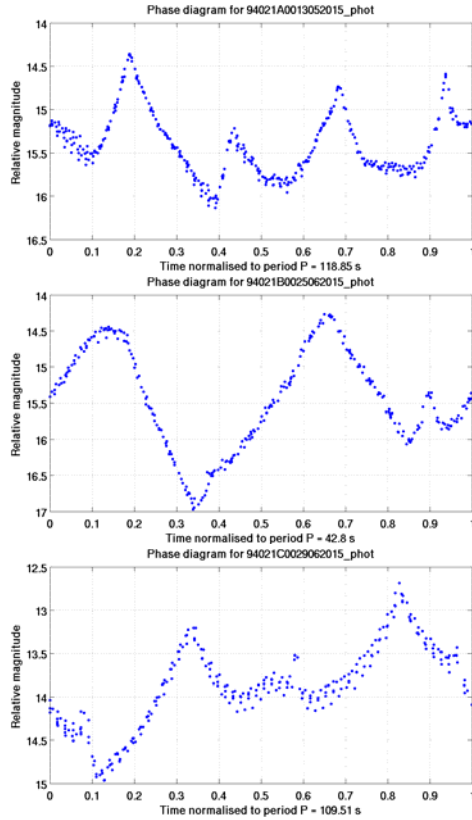


Fig. 6: Phases reconstructed from light curves and extracted apparent spin periods for objects 94021A, 94021B and 94021C. Light curves acquired by ZIMLAT during nights 2015-05-13, 2015-06-25 and 2015-06-29.

SLR measurements to objects equipped with so-called retro-reflectors (corner cubes) allow to determine the spin rates of such targets. For several years The Space Research Institute (Institut für Weltraumforschung, IWF) in Graz, Austria performs with its own SLR station the SLR measurements to cooperative and recently also to non-cooperative targets in order to determine their attitude states². Within the last year, IWF performed also SLR ranges measurements to dozen of GLONASS satellites and extracted spin periods for them. The IWF provided to AIUB some of these values, namely for 89001B, 94021A, 94021B, and 94021C for cross-confirmation purposes¹⁴. For GLONASS 89001B the value was determined to be 9.47 s for the date 2014-11-16 and 9.12 s for the date

2015-01-16. These values are in good agreement with value determined from light curve by AIUB. This the case also for other four mentioned satellites. All the values extracted by AIUB and IWF are listed in Tab. 2.

Object	Date	Period [s]	Period [s]
		AIUB light curve	IWF SLR ranges
89001B	2014-11-16		9.47
89001B	2014-12-22	9.26	
89001B	2015-01-16		9.12
94021A	2015-04-19		122
94021A	2015-05-13	118.85	
94021A	2015-06-28		118
94021B	2015-03-18		43.5
94021B	2015-06-25	42.8	
94021C	2015-06-29	109.51	
94021C	2015-07-21		109

Table 2: Summary of the apparent spin periods extracted from AIUB light curves and from IWF SLR ranges.

GEO object

The GEO object PAKSAT 1 (19960-06A) is an example of box-wing type of spacecraft which is not anymore attitude stabilised. This satellite was observed for longer period of time by M. A. Earl and G. A. Wade⁶ and once by AIUB in November 2014. Its light curve from 2014-11-07 and reconstructed phase are plotted in Fig. 7. Already by eye it is visible that only two whole periods are present in the light curve. As expected due to the small ratio between time series length and apparent spin rate the results from power spectra from Fourier based methods were un-conclusive.

The Epoch Folding method as well as the Lomb-Scargle periodogram were able to detect period values which could be confirmed by the phase reconstruction. Finally, the value of 582 s was determined as the apparent spin period for this target.

As already mentioned, PAKSAT 1 was also observed by M. A. Earl and G. A. Wade in 2013 and 2014⁶. This object showed a constant change in the apparent spin period during the campaign. The first measurement was taken at day 290 after the 2013-01-01. The estimated period was then 250 s. The longest period of 640 s was found for day 460 after the 2013-01-01. Their last measurement was taken at the 600th day after the 2013-01-01, where a spin period of 370 s was measured. All mentioned values, including the one measured by AIUB are plotted in Fig. 8. This figure was taken from⁶ and modified. As it is visible in Fig. 8 the apparent spin for this satellite is alternately accelerating and decelerating.

HAMR object

The object E07047A, which is an object of the internal AIUB catalogue was discovered in spring 2007 during

ESA/AIUB GEO surveys¹⁵. The area-to-mass ratio of E07047A was estimated during the orbit improvement to be around $4.8 \text{ m}^2/\text{kg}$ and is therefore considered as so-called High-Area-to-Mass Ratio (HAMR) object. Light curves were acquired in three different years on four different dates: on 2008-02-24, on 2008-05-07, on 2010-08-09, and on 2011-10-17.

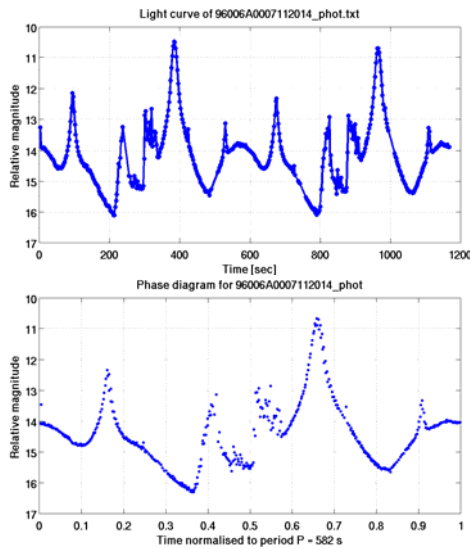


Fig. 7: Light curve (upper figure) and its reconstructed phase (lower figure) for object 96006A observed in 2014-11-07 by ZIMLAT.

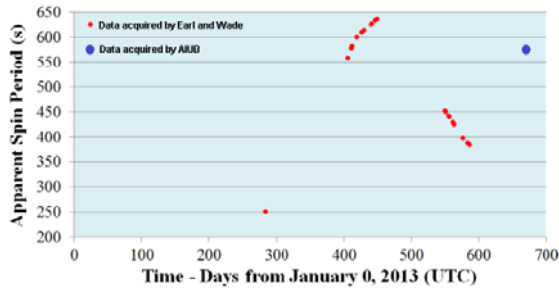


Fig. 8: Evolution of the apparent spin period measured by Earl and Wade (red circles) and by AIUB (blue circle). Figure taken from⁶.

The period output of the Fourier based method, Lomb-Scargle and Epoch folding led to the apparent period confirmed and refined by the phase reconstruction method. These values varied from 56.2 s to 94.5 s. All results are summarized in Tab. 3. The Fig. 9 is shown the extracted apparent spin period variation over time. Object E07047A seemed to slowly decrease its period over time, which means its rotation was accelerating between years 2008 and 2011.

In Fig. 10 are plotted three phase diagrams for object E07047A constructed by using apparent spin periods

equal to 94.4 s, 73.7 s and 56.2 s. Light curves were acquired by ZIMLAT at 2008-05-07, 2010-08-09 and 2011-10-17, respectively. The structure of the light curve of object E07047A seemed to be consistent during investigated time interval of 3.5 years. As it can be seen in Fig. 10, specially in the middle figure, the structure always consists from one dominant peak and three smaller side peaks.

Date	FFT	Epoch folding	Lomb-Scargle	Phase rec.
2008-02-24	48.5	unav.	unav.	94.5
2008-05-07	45.3	94	236.7,47.4	94.4
2010-08-09	76.8,16.8	73	72.9,18.5	73.7
2011-10-17	46.5	58	14,28	56.2

Table 3: Overview of the analysed light curves of E07047A, with selected periods from the power spectra.

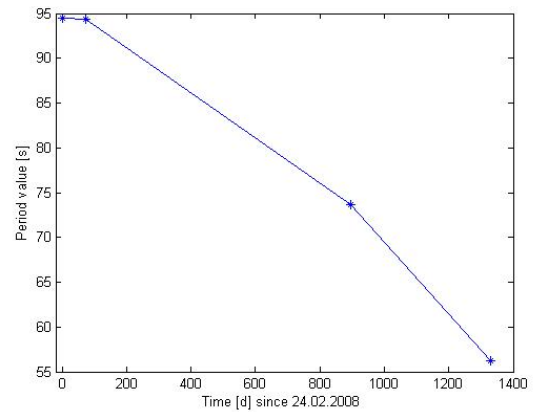
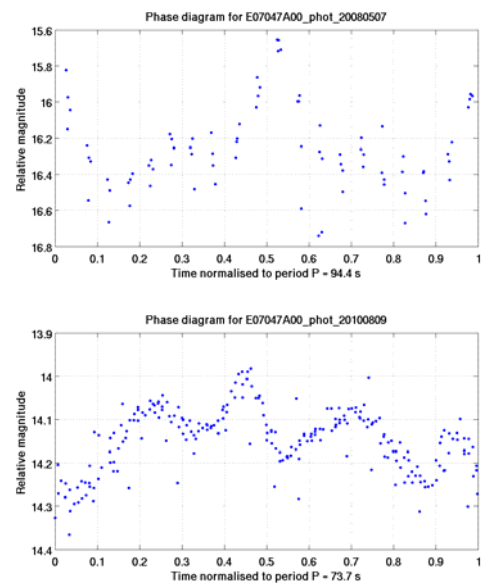


Fig. 9: From light curves extracted apparent spin period for object E07047A as a function of time.



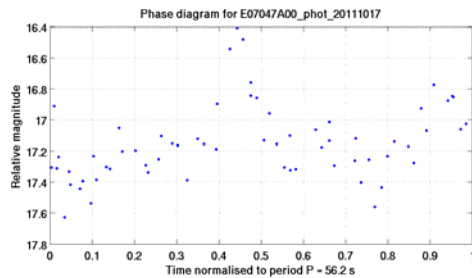


Fig. 10: Phases reconstructed from light curves and extracted apparent spin periods for object E07047A over time. Light curves acquired by ZIMLAT during nights 2008-05-07, 2010-08-09 and 2011-10-17.

V. SUMMARY AND CONCLUSIONS

AIUB is currently operating three different optical systems situated in Zimmerwald Observatory (Switzerland) in order to perform photometry and astrometry of space debris objects. The ZIMLAT telescope with 1-m aperture is used for routine light curve acquisition. For the CCD light curves observations, a fully automated pipeline including the automated active object tracking, image acquisition, and the image processing and extraction of object intensities, is available. Currently, AIUB's light curve database contains more than 1,300 light curves from which only around 10% were processed, their apparent spin rates extracted and their phase reconstructed.

Light curve acquisition and processing has a long history. There are several applications for light curves, namely to determine the attitude state of satellites and space debris defined by tumbling/stable behaviour and by the spin axis direction. Reconstructed phases can be used for modelling purposes in case that forward modelling is used or they can be applied during the object recognition/correlation process. AIUB performed detailed investigation on different types of methods usually used by astronomical community for time series analysis. We discussed these methods and their possible application for the light curves processing. Along with the Fourier based methods, like FFT, Periodogram and Welch's method, AIUB is currently using also Epoch folding, Lomb-Scargle periodogram and it's the phase reconstruction method in order to reliably determine the apparent spin rates for tumbling space objects. Based on the mentioned methods AIUB created a semi-automated tool for light curve processing, which decreased the processing time for one object to few minutes. Currently, a possibility to use only the automatized phase reconstruction method as a single method to extract apparent spin rates is investigated.

Several cases are discussed in the paper, including large population of the decommissioned GLONASS satellites, and their light curves or their reconstructed phases are given. Extracted values, as well as evolution of the apparent spin rates of an AIUB's object with high-area-to-mass ratio are shown.

¹ Santoni F., Cordelli E., Piergentili F., *Determination of Disposed-Upper-Stage Attitude Motion by Ground-Based Optical Observations*, Determination of Disposed-Upper-Stage Attitude Motion by Ground-Based Optical Observations, 10.2514/1.A32372

² D. Kucharski et al., *Attitude and Spin Period of Space Debris Envisat Measured by Satellite Laser Ranging*, IEEE Transactions on Geoscience and Remote Sensing, 2014 Dec, v. 52, p. 7651-7657, 10.1109/TGRS.2014.2316138.

³ Kanzler R., Silha J., Schildknecht T., Fritsche B., Lips T., Krag H., *Space debris attitude simulation - iOTA (In-Orbit Tumbling Analysis)*, Proceedings of AMOS Conference, Maui, Hawaii, 2015.

⁴ Silha J., Linder E., Hager M., Schildknecht T., *Optical Light Curve Observations to Determine Attitude States of Space Debris*, Proceedings of 30th International Symposium on Space Technology and Science, Kobe-Hyogo, Japan, 2015.

⁵ Larsson S., *Parameter Estimation in Epoch Folding Analysis*, Astronomy and Astrophysics, 1996, Supplement series 117, 197-201.

⁶ M.A. Earl and G.A. Wade, *Observation and Analysis of the Apparent Spin Period Variations of Inactive Box-wing Geosynchronous Resident Space Objects*, Proceedings of 65th International Astronautical Congress, Toronto, Canada, 2014, IAC-14-A6.9.1.

⁷ Veniaminov S., Dicky V., Tretyakov Yu., *The New Approach to Revealing Latent Periodicities in Radar and Optical Signatures of Space Objects*, Proceedings of 6th US/Russian Space Surveillance Workshop, St. Petersburg, Russia, 2005.

⁸ Papushev P., Karavaev Yu., Mishina M., *Investigations of the evolution of optical characteristics and dynamics of proper rotation of uncontrolled geostationary artificial satellites*, Advances in Space Research, 2009, Volume 43, Issue 9.

⁹ M. Ghil et al., *Advanced spectral methods for climate time series*, Review of Geophysics, 2002, Volume 40.

¹⁰ D. A. Leahy et al., *On searches for periodic pulsed emission: the Rayleigh test compared to Epoch folding*, The Astrophysical Journal, 1983, 1983ApJ...272..256L.

¹¹ C. Früh and T. Schildknecht, *Analysis of observed and simulated light curves of space debris*, Proceedings of the 61st International Astronautical Congress, 2010, IAC-10-A6.1.9.

¹² Blender, <http://www.blender.org/>.

¹³ D. Arnold et al., *CODE's new solar radiation pressure model for GNSS orbit determination*, Journal of Geodesy, August 2015, Volume 89, Issue 8, pp 775-791, 10.1007/s00190-015-0814-4.

¹⁴ Georg Kichner, The Space Research Institute in Graz, Austria, personnel communication, September 2015.

¹⁵ Schildknecht, T., Musci R., Ploner M., Beutler G., Flury W., Kuusela J., J. de Leon Cruz, L. de Fatima Dominguez Palmero, *Optical Observations of Space Debris in GEO and in Highly-eccentric Orbits*. Advances in Space Research, 34, 2004, pp. 901-911.