DEPENDENCE OF ORBIT DETERMINATION ACCURACY ON THE OBSERVER POSITION

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ABSTRACT

The Astronomical Institute of the University of Bern (AIUB) is conducting several search campaigns for space debris in Geostationary (GEO) and Medium Earth Orbits (MEO). Usually, to improve the quality of the determined orbits for newly discovered objects, follow-up observations are conducted. The latter take place at different times during the discovery night or in subsequent nights. The time interval between the observations plays an important role in the accuracy of the calculated orbits. Another essential parameter to consider is the position of the observer at the observation time. In this paper, the accuracy of the orbit determination with respect to the position of the observer is analyzed. The same observing site at varying epochs or multiple site locations involve different distances from the target object and a different observing angle with respect to its orbital plane and trajectory. The formal error in the orbit determination process is, among other dependencies, a function of the latter parameters. The analysis of this dependence is important to choose the appropriate observation strategy. One of the main questions that arises is e.g. whether observing the same object from different stations results in better determined orbits and, if yes, how big is the improvement. Another question is e.g. whether the observation from multiple sites needs to be simultaneous or not for a better orbit accuracy.

1. INTRODUCTION

The Astronomical Institute of the University of Bern (AIUB) is conducting optical search campaigns for high altitude objects using the ESA Space Debris Telescope (ESASDT) on Tenerife on behalf of ESA. The aim of these campaigns is to improve the statistical information about the populations of objects in Geostationary Orbits (GEO) [1], Geostationary Transfer Orbits (GTO) [2], and Medium Earth Orbits (MEO) [3]. A large amount of faint and unknown objects, as well as a new population of objects with a very high area-to-mass ratio have been observed within these surveys [4]. In general only a short observation arc is available for most of these objects. These short arcs do not allow determining an accurate full six parameter orbit. Normally, circular orbits are determined instead. A circular orbit is a good approximation for GEO, but not for eccentric orbits like GTO. Possible concepts for a catalogue of objects were developed in the framework of ESA studies for a European Space Surveillance Network [5][6]. AIUB participated in these studies, where the work focused on the selection of optical detectors, the development of survey strategies for high-altitude orbits, and on the performance estimation. According to the developed concepts, to improve the quality of the determined orbits for newly discovered objects, follow-up observations are conducted. Since the discovery track of an object usually consists of a small number (two to ten) of observations and the track length is only a few minutes, follow-up observations are needed in order to get a longer observation arc. Follow-ups from several nights are needed if the orbit should be accurate enough to be included into a catalogue. Several studies have investigated the optimal sequence of follow-ups and the time intervals between subsequent observations to achieve the best orbit accuracy [7][8]. From the investigations it resulted that e.g. for GEO at least two follow-up tracks are necessary to recover a discovered object during the following night. The ideal time interval between the tracks was found to be one hour. This allows recovering the object with the small field of view (FOV) of 0.7” at the ESASDT. For these strategies the observations from only one site were considered. However, AIUB participated also to joint observations of GEO objects performed by several astronomical observatories. The program is lead by the Russian Academy of Science (RAS). AIUB is contributing to this program using its own 1-meter telescope in Zimmerwald (ZIMLAT). The aim of the program is to continuously track recently discovered unknown objects over a longer time frame. Test campaigns for acquiring simultaneous optical observations from two sites were performed and the benefit of observations from multiple sites compared to observations from one site was investigated using simulations [9]. In both cases, observing from one or multiple sites, the geometry of the observation is relevant for the accuracy of the orbit determination. The geometric factors essentially comprise the distance from the station to the object and the angle between the line of sight and the trajectory of the object. In this work the dependence of the accuracy in the orbit determination on these parameters is investigated. The question of the simultaneous observation from different sites is then addressed based on the results of the analysis.

2. OBSERVATION GEOMETRY

The geometry considered in the analysis of the problem is illustrated in Figure 1and Figure 2. For the moment, in this study, only circular orbits in two simple geometric situations are examined. In one situation the orbit plane coincides with the Earth equatorial plane (Figure 1). The angle α describes the geocentric difference in right ascension of the object in the positions C and D. In the case of one station A the object can be observed at the zenith (position C) or later at the position D. In the case of two stations the object in C is observed simultaneously from A and B. Figure 2 illustrates a different situation where the orbit plane is inclined at an angle δ with respect to the equatorial plane. The object in C is observed from the station A, while a station in B would observe it at the zenith.

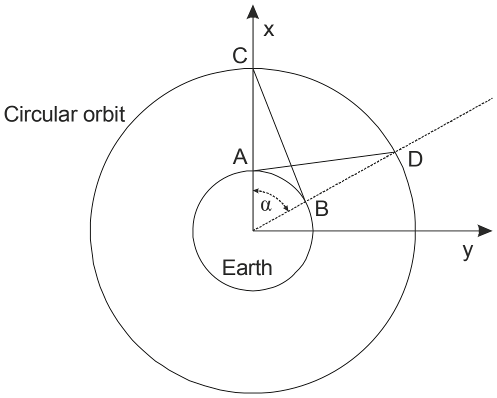

Figure 1. Geometry with circular orbit in the equatorial plane. The angle α indicates the difference in longitude or right ascension.

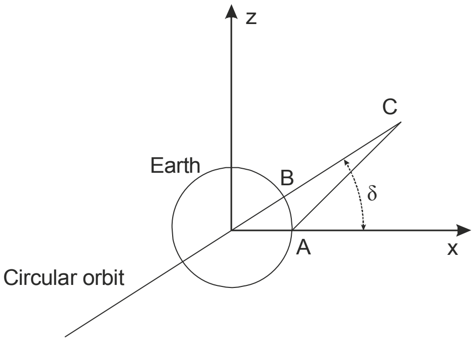

Figure 2. Geometry with circular orbit inclined at an angle δ with respect to the equatorial plane.

3. SIMULATIONS

For the simulations circular orbits with three different semimajor axes representative for GEO, MEO, and Low Earth Orbit (LEO) orbits were selected: 8000 km, 26000 km, and 42000 km. Ephemerides of objects in the three different orbits were calculated and topocentric observations from the stations A and B were simulated. In all the simulations, if not explicitly indicated, a mean astrometric error of 0.5” was assumed for the accuracy of the single observation and the tracklets consist of three observations within a 15 s arc length. The initial orbit determination was performed using the “Celmech” software environment developed at AIUB [10].

Analysis of semimajor axis

The analysis of the orbit determination accuracy in the geometry described in Figure 1 is limited to the semimajor axis. The latter, if only circular orbits are assumed, is representative of the degree of accuracy achieved in the orbit determination process. Furthermore, in the considered geometry, for symmetry reasons, the observations of D from A and C from B are equivalent and lead to the same results. Therefore in the subsequent analysis in this section only the case observing from A will be examined. Figure 3 shows the formal error Δa in the semimajor axis as a function of the angle α for orbits in LEO, MEO, and GEO. The formal error is mainly dependent on two distinct components. On one hand there is the observation error: relevant for our considerations is the error Δα regarding the topocentric measured position in right ascension. On the other hand the accuracy depends on the length of the observed arc and the number of observations. The influence of Δα in the formal error Δa can be estimated using geometric considerations. Figure 4, Figure 5, and Figure 6 show for LEO, MEO, and GEO, respectively, the observation error $\Delta\alpha_{geo}$ at the geocenter as a function of the angle α for different error values Δα indicated in the color bar. The error at the geocenter is calculated propagating the measurement error Δα in the transformation formula from topocentric to geocentric coordinates. After this transformation the orbit determination in the simulations can be performed considering observations with error $\Delta\alpha_{geo}$ from a hypothetical station at the geocenter. Obviously the error Δa is then proportional to $\Delta\alpha_{geo}$ as also shown in Figure 7 for LEO, MEO, and GEO orbits. As mentioned before, the proportionality depends on the arc length and the number of observations. In Figure 8 the error Δa as a function of the arc length is plotted for the three types of orbit. Here three observations within the arc and $\Delta\alpha_{geo}$ = 0.5” are assumed. Figure 9 displays the dependence of the error Δa from the number of observations. For these simulations 60 s arc length and $\Delta\alpha_{geo}$ = 0.5” are considered.

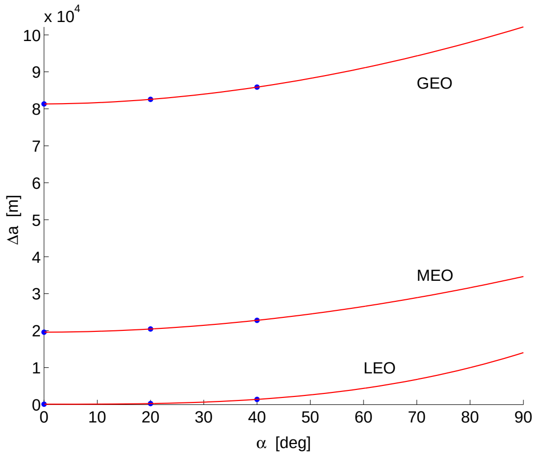

Figure 3. Formal error Δa in the semimajor axis as a function of α for LEO, MEO, and GEO orbits.

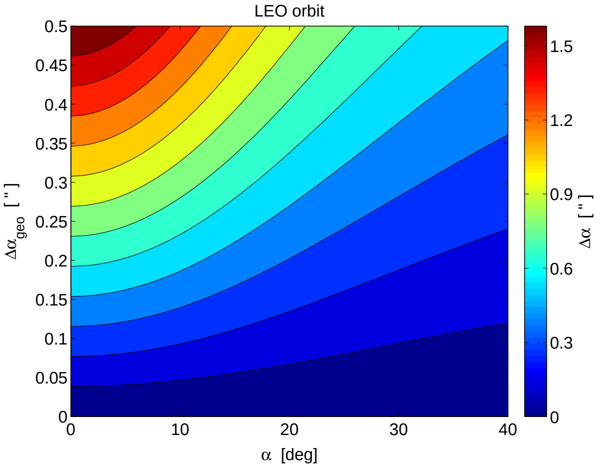

Figure 4. Error $\Delta\alpha_{geo}$ at the geocenter vs. angle α for different error values Δα for orbits in LEO.

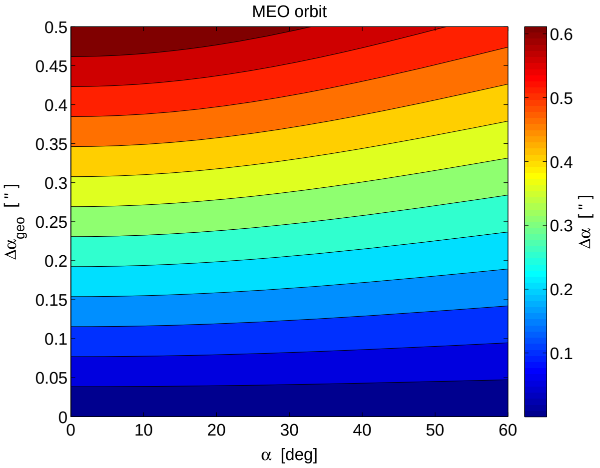

Figure 5. Error $\Delta\alpha_{geo}$ at the geocenter vs. angle α for different error values Δα for orbits in MEO.

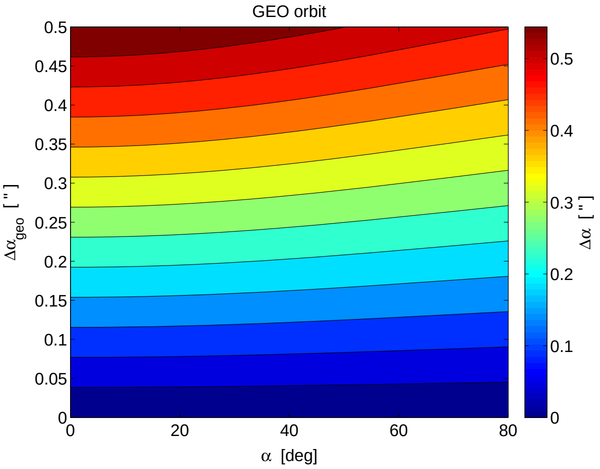

Figure 6. Error $\Delta\alpha_{geo}$ at the geocenter vs. angle α for different error values Δα for orbits in GEO.

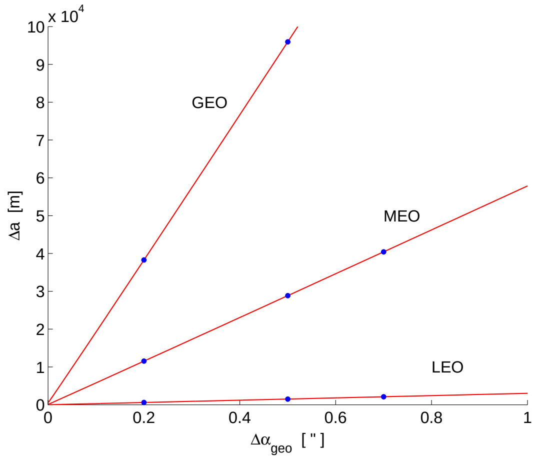

Figure 7. Formal error Δa as a function of $\Delta\alpha_{geo}$ for LEO, MEO, and GEO orbits.

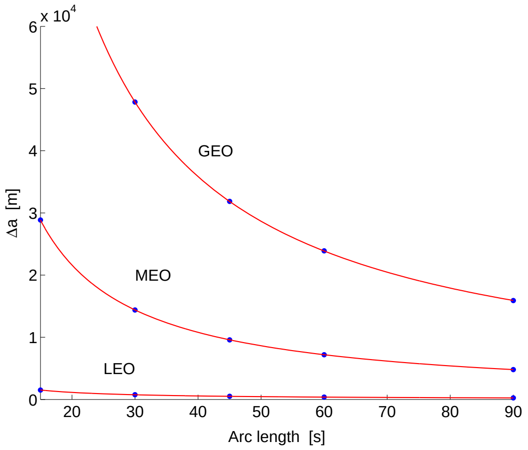

Figure 8. Formal error Δa as a function of the arc length with $\Delta\alpha_{geo}$ = 0.5" and three observations within the arc for LEO, MEO, and GEO orbits.

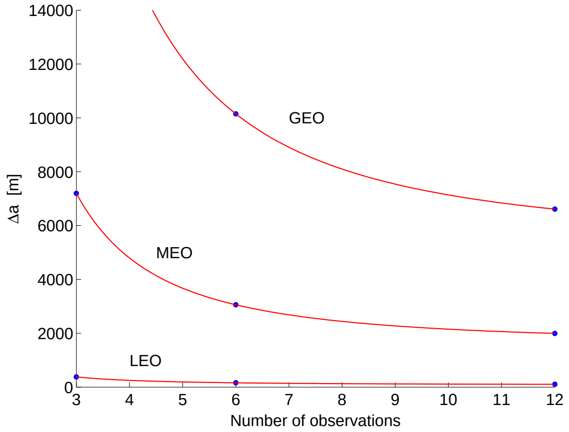

Figure 9. Formal error Δa vs. number of observations with $\Delta\alpha_{geo}$ = 0.5” in 60 s arc length for LEO, MEO, and GEO orbits.

Analysis of inclination

In the situation of Figure 2 the analysis is limited to the accuracy of the inclination after the orbit determination. In this geometry the orbit inclination parameter is a good candidate to express the quality of the orbit determination. Figure 10 illustrates the error Δi as a function of δ for the three reference orbits. Similarly to the case with α the formal error Δi in the inclination has a geometric dependence on δ and the topocentric error Δδ can be reduced to $\Delta\delta_{geo}$ at the geocenter. Hence, the dependence of Δi on the number of observations can be simulated considering observations with error $\Delta\delta_{geo}$ from a hypothetical station at the geocenter. Figure 11 shows these simulations with $\Delta\delta_{geo}$ = 0.5” and 60 s arc length.

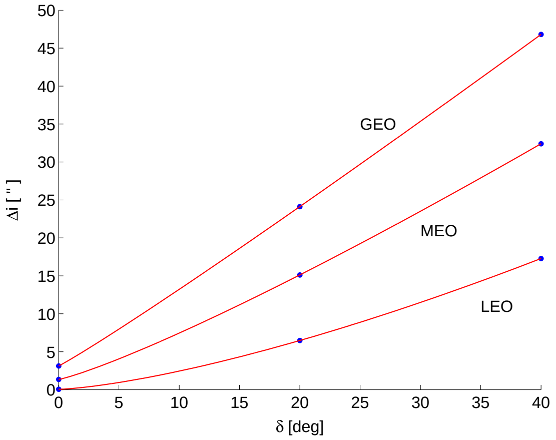

Figure 10. Formal error Δi in the inclination as a function of δ for LEO, MEO, and GEO orbits.

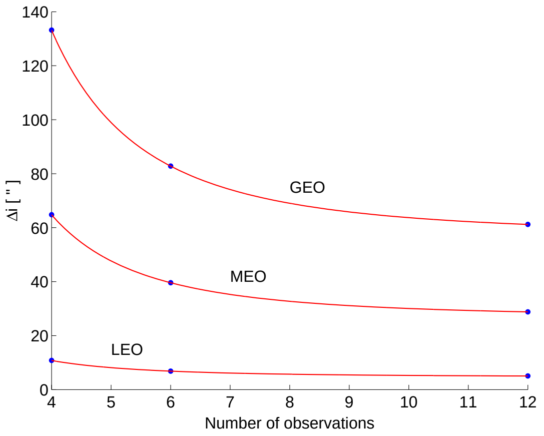

Figure 11. Formal error Δi vs. number of observations with $\Delta\delta_{geo}$ = 0.5” in 60 s arc length for LEO, MEO, and GEO orbits.

Observations from two sites

Simulations of simultaneous observations from two different stations were conducted. The stations have a difference in longitude α according to Figure 1. As shown in Figure 12 there is a clear improvement observing from two stations compared to observations from one station only. The latter case corresponds to α = 0 deg in the plots. For all angles α the considered arc length is 30 s and six observations are simulated. The scenario with observations from two sites at different times, e.g. observing C from A and later D from B (see Figure 1), was simulated as well. In the latter case it is possible to minimize α and δ choosing the appropriate time of the observation and, as identified above, smaller α and δ result in better orbit determination accuracy.

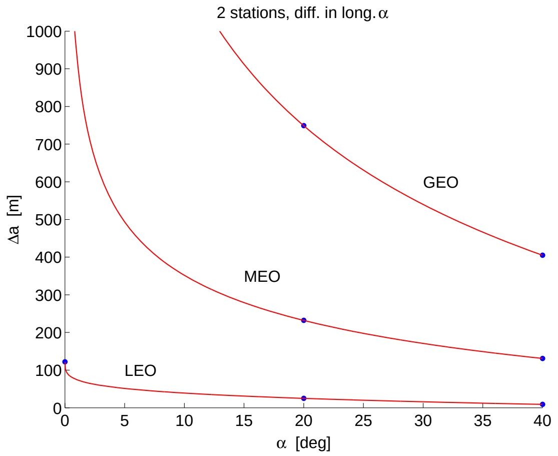

Figure 12. Formal error Δa vs. longitude difference of the two stations. Assumed are 30 s arc length and six observations.

4. CONCLUSIONS

For the two examined situations with simple geometries described by the angles α and δ, the errors Δa and Δi are given by distinct contributions according to the position of the object relative to the stations, the length of the observed arc, and the number of observations. The contribution regarding the relative position can be calculated from the topocentric errors $\Delta\alpha$ and $\Delta\delta$, given the angles α and δ. The topocentric errors are reduced to errors $\Delta\alpha_{geo}$ and $\Delta\delta_{geo}$ at the geocenter and the influence of arc length and number of observations can be evaluated in the geocentric geometry. Simultaneous observations from two stations substantially improve the accuracy of the orbit determination. However, taking into account the error dependence on α and δ, the optimal condition is the observation of the same object from two stations at different times optimized for small α and δ.

5. REFERENCES

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