# Comparison of different Methods of Ephemeris Retrieval for Correlation of Observations of Space Debris Objects 

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#### Abstract

1 ABSTRACT In the observation of space debris objects so called tracklets, short series of astrometric positions spanning an interval of a few minutes, are gained. Those are correlated with external catalogues like DISCOS, which are given in the two line elements format, and with internal catalogues. Since the tracklets do not allow to determine a full six parameter orbit, positions and velocities are correlated with ephemeris. The accuracy of the ephemerides retrieved from different catalogues differ tremendously, the differences are discussed. The offset between predicted and observed positions of objects is also relevant in the context of space situation awareness and collision avoidance.


## 2 INTRODUCTION

The Astronomical Institute of the University of Bern (AIUB) performs since several years observations of the space debris populations in the geostationary ring (GEO) and in geostationary transfer orbits (GTO) and other high eccentricity orbit using two 1 m telescopes, the ESA Space Debris Telescope (ESASDT) on Tenerife, Canary Island, and the ZIMmerwald Laser and Astrometry Telescope (ZIMLAT), located close to Bern, Switzerland.

One important task is to perform an exact prediction of the positions of objects that are to be observed. On the one hand to plan so called follow-up observations and, on the other hand, to identify the observed objects and distinguish it from possibly many observed objects on the same frames. Furthermore precise predictions are essential in collision avoidance.

When observing space debris objects, so called tracklets, short series of astrometric positions, are gained spanning an interval of a few minutes. They do not allow to determine a full six parameter orbit. Thus a correlation with a catalogue using orbital elements is impossible. Therefore the position and velocity information is compared with ephemerides, stemming from propagated orbits. Two different kinds of ephemerides are discussed. Ephemerides retrieved from orbital elements distributed in the so called DISCOS catalogue in the two line element format (TLE) and ephemerides retrieved from osculating elements calculated on the basis of AIUB observations.

## 3 ERRORS INTRODUCED BY METHODOLOGY

When tracklets are gained, only the information about the direction of observations is available in optical observations, information about the radial distance is only available through orbit determination. No full six parameter orbit can be determined using a single tracklet. To be able to compare catalogue data with tracklets for example when processing observation data in real time, the radial distance of the observed object has to be estimated. A favourable method to estimate the radial distance is to assume that the radial distance of the observed object is the same as the one of the correlated catalogue object. The offset between the two positions is then determined as the angular distance in degrees on the celestrical sphere and as a projected intrack and projected crosstrack direction (NTW system, see [1] for details) in the tangent plane with the origin in the catalog object's position on the celestrical sphere. More details on this and other methods to estimate the radial distance can be found in [2].

The residuals between observed and expected position with the estimation of radial distance as mentioned above has been studied in an artificial setup of a pseudo-observed object at inclination zero and a semi-major axis of 26000 km . The pseudo-observed object is shifted intrack:crosstrack:radial $60 \mathrm{~km}: 20 \mathrm{~km}: 20 \mathrm{~km}$ in position in NTW system relative to a


Figure 1: residuals in intrack, crosstrack and radial direction (NTW system) as a function of true anomaly (a) at eccentricity zero, (b) eccentricity $0.24,(c)$ eccentricity $0.5,(d)$ and eccentricity 0.75.


Figure 2: residuals in intrack, crosstrack direction projected in tangent plane as a function of true anomaly (a) at eccentricity zero, (b) eccentricity 0.24 , (c) eccentricity 0.5 , (d) and eccentricity 0.75.
pseudo-catalogue object position. In Fig. 1 the residuals in intrack, crosstrack and radial direction in NTW system under the assumptions mentioned above are shown as a function of the true anomaly observed from the topocentric position of Tenerife at hour angle zero for different values of eccentricity. In Fig. 2 the residuals in the projected intrack and crosstrack direction are shown. In both cases the residuals are determined in percent of the true values.

As shown in Fig. 2 the projected intrack residuals are nearly constant over all eccentricities and anomalies, the value is close to 40 percent. The projected crosstrack residuals show larger variations especially with higher eccentricities. They reach up to $\pm 200$ percent for anomalies close to zero resp. 360 degrees and at 180 degrees. At 180 degrees the sign flips due to the changing orientation of the projection plane.

## 4 COMPARISON OF OBSERVATIONS WITH EPHEMERIS

With the method mentioned in the previous section observation tracklets have been compared with ephemerides. The ephemerides have been retrieved on the one hand from TLEs stemming from the DISCOS catalogue, on the other hand, from of orbits, that were determined from prior observations of the objects. Validation that the new observation tracklets in fact belong to the correlated object was performed by correlation with TLEs from DISCOS, using the correlation algorithm described in [2] and by a successful orbit determination including the new observation tracklets.

Orbit determination was performed with a slightly modified version of the program tool CelMech, described in [3]. First a six-parameter orbit is determined from two or more tracklets stemming from the same night. The force field, which is used, includes the third body perturbations due to Sun and Moon and the oblateness of the Earth. In a second step the orbit is improved by including more observations over several days. An improved force field is used in this step, including Earth's potential coefficients up to order and degree 12, perturbations due to Earth tides, corrections due to general relativity and a simple model for estimating the direct radiation pressure (DRP). Earth shadow is modelled.

The simple model for DRP works with a predefined value for the area to mass ratio, it is fixed at the value $0.02 \mathrm{~m}^{2} \mathrm{~kg}^{-1}$, which is the value for a standard GPS satellite. The reflection coefficient is fixed at the value 2.0 , which means, that all radiation is absorbed (in contrary to other definitions, as for example in [1]). The Earth is assumed to be spherical and the satellite to be rotationally invariant.

In addition a more sophisticated model for the radiation pressure can be used. In this model the area to mass ratio is estimated as a DRP scaling factor in addition to the osculating elements. In the advanced estimation of DRP earths'
oblateness is taken into account for shadow computations as well as Earth's re-radiation. In addition so called biases can be estimated. Biases are perturbations, which may be used to account for asymmetries in the observed object, e.g. misalignment of solar panels in case of satellites. Biases are estimated in direction of the axes in the traditional radial, along and crosstrack (RSW) system independently from each other and can be used in addition to both radiation pressure models. For more details consult [4]. The area to mass ratio is assumed to be constant. The estimation of biases not only has effects on the estimation of osculating elements but also on the estimated area to mass ratio, of course. For the generation of ephemerides the osculating elements are propagated with the improved force field under consideration of the estimated or predetermined area to mass ratio and the reflection coefficient. The estimated biases are not available for orbit propagation.

To derive ephemerides from TLEs from DISCOS the well known SDP8 model is used to propagate the orbits (for details see [5]). For DISCOS TLEs no information is available about the last observation epoch at which a new orbit has been determined; therefore this could not be taken into account in the following analysis.

A few representative examples for correlation of ephemerides of propagated orbits with observations for three different object classes are shown in this paper. The first class consists of GEO objects, the second of GTO objects and objects in Molniya orbits. The last class consists of high area to mass ratio objects. The examples chosen for the first two classes can be found in the DISCOS catalogue, the examples for the last class stem from the internal catalogue of the AIUB, unfortunately no official TLEs are available for them.

All observations that were used to determine the orbits as well as the observations used for comparison with ephemerides stem from the two 1 m telescopes, ESASDT and ZIMLAT. The objects, which can be found in the DISCOS catalogue (classes GEO and GTO/Molniya), were observed with ZIMLAT over the past four years. The objects which are not contained in the DISCOS catalogue (class high area to mass ratio objects) are observed with both the ESASDT and ZIMLAT since 2001.

### 4.1 Examples for Geostationary Objects

As representative examples the three GEO objects 79105A (Gorizont-3), 83089B (Insat-1B) and 80081A (Raduga-7) have been investigated in detail. All have vanishing eccentricities and inclinations between 12 and 14 degrees. In Fig. 3 the angular distances between the observed and calculated positions are shown. The computed positions were either derived from TLEs or from orbit determination. The epoch of the last observation, which entered orbit determination, is in all cases close to the date at which the comparison between observations and ephemerides starts. The epoch of the last observation is unknown for TLEs, as mentioned above.

In Fig.3a the residuals for ephemerides from TLEs and from three different orbit determinations are shown for the object 79105A. One orbit was determined with a ten day observation arc with the simple DRP model (EPHMS, no additional parameters, $\mathrm{A} / \mathrm{M}$ fixed at $0.02 \mathrm{~m}^{2} \mathrm{~kg}^{-1}$, $\mathrm{rms}=0.41$ "). The second and third orbit were determined with a 32 days arc first with the simple DRP model ( $E P H M_{-}$, no additional parameters, A/M fixed at $0.02 \mathrm{~m}^{2} \mathrm{~kg}^{-1}$, rms=1.11") and then with the advanced DRP model. An area to mass ratio of $0.0069 \mathrm{~m}^{2} \mathrm{~kg}^{-1}$ was estimated in the latter case (EPHM_LDRP, $\mathrm{rms}=0.48$ ").

In Fig. 3 b the residuals of five different kinds of ephemerides are shown for the object 83089B. Besides the TLE ephemerides residuals, the residuals of four different orbit determinations are shown. An orbit was determined with a very short arc of only two days (EPHM_VS, no additional parameters, A/M fixed at $0.02 \mathrm{~m}^{2} \mathrm{~kg}^{-1}, \mathrm{rms}=0.17$ "), with a short arc of nine days


Figure 3: Angular distance between the observed and calculated ephemerides of TLEs and orbit determination for the GEO objects (a) 79105A, (b) 83089B, (c) 80081A.


Figure 4: Residuals between observed positions and calculated ephemerides from TLEs and from orbit determination for GEO object 79105A


Figure 5: Residuals between observed positions and calculated ephemerides from TLEs and from orbit determination for GEO object 83089B
( $E P H M_{-}$, no additional parameters, $\mathrm{A} / \mathrm{M}$ fixed at $0.02 \mathrm{~m}^{2} \mathrm{~kg}^{-1}$, $\mathrm{rms}=0.40$ "), with a long arc of 23 days ( $E P H M \_$, no additional parameters, A/M fixed at $0.02 \mathrm{~m}^{2} \mathrm{~kg}^{-1}$, $\mathrm{rms}=1.04$ ") and with the same arc but with the advanced DRP model ( $E P H M \_L D R P, \mathrm{~A} / \mathrm{M}=0.0273 \mathrm{~m}^{2} \mathrm{~kg}^{-1}, \mathrm{rms}=1.04$ ").

Fig.3a and Fig.3b clearly show, that ephemerides of the short arc orbit determinations result in large residuals. But in the case of the object 79105A a short arc of two days and in the case of the object 83089B a short arc of nine day already has smaller residuals than the TLE ephemerides. The best solutions are in both cases the orbits determined with the long arc and the advanced DRP model. Fig. 4 and Fig. 5 show the residuals of the ephemerides from TLEs and from both long arc orbit determinations for both objects in more detail. For both objects the residuals of ephemerides generated with the advanced DRP model are nearly constant within the time interval of 40 days since orbit determination, whereas the residuals of the TLE ephemerides show large variations. The angular distance for the object 79105A is still below 0.0007 degrees and for object 83089B even below 0.0003 degrees within the first 30 days since orbit determination, the residuals for the TLEs are as large as 0.06 degrees for the same epochs. The TLE intrack and crosstrack values are in


Figure 6: Residuals between observed positions and calculated ephemerides from TLEs and from orbit determination for GEO object 80081A.
the range of 40 km and 15 km , respectively. For the orbit determinations the intrack residuals are below 0.1 km and the crosstrack residuals below 0.2 km for both objects within the first thirty days since orbit determination. The behaviour of the residuals of the angle between direction of velocity is very similar for the TLE ephemerides and the ones from orbit determination. The direction of velocity is strongly correlated with the orbital plane of the object.

In Fig.3c the residuals for the object 80081A are displayed. Besides the residuals for the ephemerides from TLEs, the residuals of a five day short arc orbit determination $\left(E P H M_{-} S\right.$, no additional parameters, A/M fixed at $0.02 \mathrm{~m}^{2} \mathrm{~kg}^{-1}$, $\mathrm{rms}=0.20$ "), of two 15 day long arc orbit determination, one with the simple radiation pressure model (EPHM_M, no additional parameters, A/M fixed at $0.02 \mathrm{~m}^{2} \mathrm{~kg}^{-1}$, $\mathrm{rms}=0.28$ ") and with the advanced DRP model (EPHM_MDRP, $\mathrm{A} / \mathrm{M}=0.0125516 \mathrm{~m}^{2} \mathrm{~kg}^{-1}, \mathrm{rms}=0.21$ ") and of two 20 day long arcs one with the simple DRP pressure model (EPHM_L, no additional parameters, A/M fixed at $0.02 \mathrm{~m}^{2} \mathrm{~kg}^{-1}$, $\mathrm{rms}=1.72$ ") and one with the advanced DRP model ( $E P H M \_L D R P$, $\mathrm{A} / \mathrm{M}=0.01275 \mathrm{~m}^{2} \mathrm{~kg}^{-1}$, $\mathrm{rms}=0.28$ "), are shown.

Object 80081A shows larger residuals than the other two objects, see Fig.3c and Fig.6. The ephemerides from TLEs again show large variations, whereas all residuals of ephemerides from orbit determinations rise fast towards larger times. Fig.3c shows that all orbit determination ephemerides residuals show a similar behaviour. This time, the orbit determinations with the advanced DRP model do not provide the best ephemerides. Although the object is in a similar orbit than the other two objects and the shape of Raduga-7 is quite similar to that of the Gorizont-3 satellite; the advanced DRP model seems not to be perfectly suitable for this satellite over longer times intervals. The orbit determination is successful for observation arcs up to 20 days, but with longer arcs no orbit determination with an acceptable root mean square is possible. One reason could be changes in attitude motion. The orbit determination with the simple DRP model based on the medium arc of 15 days shows the smallest residuals. Fig. 6 shows the residuals of the best solutions in more detail. It can be seen, that the crosstrack component shows a secular trend with large residuals even within the first days since orbit determination. 22 days after orbit determination the angular distance is even though the orbit does not seem to be optimally determined below 0.003 degrees. The largest intrack and crosstrack values are 1.3 km and 2.1 km respectively. But when the ephemerides are compared to observations 33 days after orbit determination the angular distance reaches already values of 0.013 degrees, the intrack and crosstrack residuals of the order of 1.7 km and 8.8 km , respectively. The difference to the best solution with the advanced DRP model is in the order of 0.002 degrees in angular distance within the first twenty days. The residuals in the angle of direction of velocity shows a nearly identical behaviour for the medium arc orbit determination ephemerides and for TLE ephemerides.

### 4.2 Examples of Objects in highly eccentric Orbits

As representatives for objects in highly eccentric orbits the following three objects have been investigated: Ariane $5 \mathrm{R} / \mathrm{B}$ rocket body 00016D, in an orbit with 6.3 degrees inclination, and two Molniya objects in orbits with inclinations of about 63 degrees: Molniya-3 77105A and Blok-ML 92085D, the latter decayed meanwhile. All object orbits have eccentricities around 0.7 ; their perigee altitude is between 8400 km and 7000 km .

In Fig. 7 the residuals in angular distance of the ephemerides from TLEs and from orbit determinations are shown. For the object 00016 D ephemerides were obtained from short arc orbit of seven days ( $E P H M_{-} V S$, no additional parameters, $\mathrm{A} / \mathrm{M}$ fixed at $0.02 \mathrm{~m}^{2} \mathrm{~kg}^{-1}$, rms $=0.28^{\prime \prime}$ ), an arc of 10 days ( $E P H_{-} V$, no additional parameters, $\mathrm{A} / \mathrm{M}$ fixed at $0.02 \mathrm{~m}^{2} \mathrm{~kg}^{-1}$, rms $=1.20^{\prime \prime}$ ) and an arc of 14 days ( $E P H M$ LBIAS, estimation of biases, simple DRP model, A/M fixed at $0.02 \mathrm{~m}^{2} \mathrm{~kg}^{-1}$, rms $=0.37^{\prime \prime}$ ). The area to mass ratio could not be estimated in this case, since orbit determination with the advanced


Figure 7: Angular distance between observed positions and calculated ephemerides from TLEs and from orbit determination for the (a) GTO object 00016D and for the objects in Molniya orbits (b) 77105A and (c) 92085D.


Figure 8: Residuals between observed positions and calculated ephemerides from TLEs and from orbit determination for the GTO object 00016D.


Figure 9: Residuals between observed positions and calculated ephemerides from TLEs and from orbit determinations for the Molniya object 77105A.

DRP model failed. For the object 77105A ephemerides were gained from TLEs and from three different orbit determinations covering an arc of 20 days. First without estimation of any DRP or biases (EPHM_L, A/M fixed at $0.02 \mathrm{~m}^{2} \mathrm{~kg}^{-1}$, rms $=0.68$ "), then with the advanced DRP model ( $E P H M \_L D R P, \mathrm{~A} / \mathrm{M}=0.00815 \mathrm{~m}^{2} \mathrm{~kg}^{-1}, \mathrm{rms}=0.28$ ") and finally with the DRP model together with additional estimation of biases (EPHM_LBIAS, A/M=0.00741 $\mathrm{m}^{2} \mathrm{~kg}^{-1}$, $\mathrm{rms}=0.17$ '). For the object 92085D residuals were determined for TLE ephemerides and for the ephemerides of three different orbit determinations. Firstly, out of an orbit covering a short arc of five days ( $E P H M_{-} V S$, no additional parameters, A/M fixed at 0.02 $m^{2} \mathrm{~kg}^{-1}$, rms $=0.35$ "), for a 49 days arc first with the simple DRP model ( $E P H M_{-} L$, no additional parameters, A/M fixed at $0.02 \mathrm{~m}^{2} \mathrm{~kg}^{-1}, \mathrm{rms}=0.81$ ") and then with the advanced DRP model ( $E P H M \_L D R P, \mathrm{~A} / \mathrm{M}=0.00508 \mathrm{~m}^{2} \mathrm{~kg}^{-1}, \mathrm{rms}=0.15$ ").

Fig.7a shows that for object 00016D the residuals of all orbit determination ephemerides show a similar behaviour. It reveals that the ephemerides of the orbit determination of the long arc including estimation of biases has slight advantages compared to the other ephemerides of the orbit determination. In Fig. 8 the residuals of this orbit determination and of the TLE ephemerides are shown in more detail. The orbit determination ephemerides residuals are a lot smaller, less than 0.006 degrees in angular distance during the first 30 days after orbit determination. The residuals of the TLE ephemerides, which are as large as 0.06 degrees, have their smallest value at 0.012 degrees. The absolute intrack value is lower than 1.4 km and the crosstrack value lower than 0.79 km within the first 30 days since orbit determination. Residuals in the angle between velocity direction are quite similar, as well in magnitude as in general behaviour. Large variations occur.

Fig. 7b reveals that for object 77105A all orbit determinations show much smaller residuals than the TLEs. The smallest residual for the TLE ephemerides has a value of 0.017 degrees, whereas the residuals of the ephemerides from the orbit determination with the advanced DRP model and biases are smaller than 0.007 degrees 45 days after orbit determination. Since there is a gap of 44 days in the observations of object 77105A, only the first couple of days since orbit determination are plotted in Fig.9; in addition no residuals for the TLE ephemerides are shown. The residuals of the solution without biases and with the simple DRP model shows large variations. The variations of the residuals of the other orbit determinations are remarkably smaller.

Object 92085D shows the largest residuals, as shown in Fig.7c, compared to the other two objects. The residuals for


Figure 10: Residuals between observed positions and calculated ephemerides from TLEs and from orbit determination for the Molniya object 92085D.
the short arc orbit determination are the largest. Fig. 10 shows the long arc orbit determination ephemerides residuals in more detail. The residuals of both orbit determinations are smaller than the ones of TLE ephemerides for the time interval up to 80 days since orbit determination. Both, the intrack and the crosstrack component residuals show large variations for both orbit determination solutions, but still smaller than the ones of the TLE ephemerides residuals. The smallest angular distance residuum of the TLE ephemerides is at 0.02 degrees. 35 days after orbit determination with the advanced DRP model the residuals in angular distance are at 0.0047 degrees, intrack at 2.7 km and crosstrack at 1.77 km in absolute values.

### 4.3 Examples of Objects with high Area to Mass Ratios

As examples for objects with high area to mass ratios, three objects of the internal AIUB catalogue EGEO07, EGEO45 and E06207B have been investigated. All objects are in more or less geostationary orbits; EGEO07's orbit has a small eccentricity and an inclination of around 16 degrees. It has an estimated area to mass ratio close to $2 \mathrm{~m}^{2} \mathrm{~kg}^{-1}$. EGEO45 is in an orbit with an eccentricity of 0.11 and an inclination close to 10 degrees, its area to mass ratio is close to $3 \mathrm{~m}^{2} \mathrm{~kg}^{-1}$. E06207B's orbit has an eccentricity of 0.43 and an inclination of 12 degrees. It has an area to mass ratio of over 30 $m^{2} \mathrm{~kg}^{-1}$. The area to mass ratios of these objects where investigated in detail in [6]. Reto Musci could show, that the area to mass ratios found by orbit determination are not constant over time.

In Fig. 11 the angular distance residuals are shown for ephemerides from different kinds orbit determinations. In addition the average value of 0.04 degrees (see [2] for further details) for TLE ephemerides of GEO or GTO objects is shown as a reference. This value is most likely not representative for high area to mass ratio objects.

For the object EGEO07 four different orbits were determined. Firstly, a short two days arc orbit was determined with the simple DRP model (EPHM_S, no additional parameters, A/M fixed at $0.02 \mathrm{~m}^{2} \mathrm{~kg}^{-1}$, $\mathrm{rms}=6.88$ ") secondly, the same arc was used to determine an orbit with the advanced DRP model and an area to mass ratio of $1.9608 \mathrm{~m}^{2} \mathrm{~kg}^{-1}$ (EPHM_SDRP, rms $=0.43$ ") was found. Thirdly, orbits were determined with an arc of 22 days with the simple DRP model (EPHM_L, no additional parameters, A/M fixed at $0.02 \mathrm{~m}^{2} \mathrm{~kg}^{-1}$, $\mathrm{rms}=76.36$ ") and with the advanced DRP model, an area to mass ratio of $1.9723 \mathrm{~m}^{2} \mathrm{~kg}^{-1}\left(E P H M \_L D R P, \mathrm{rms}=0.98\right.$ ") was estimated.


Figure 11: Angular distance between observed positions and calculated ephemerides from orbit determination for the high area to mass ratio objects EGGEO07, EGEO45 and E06207B


Figure 12: Angular distance between the observed and calculated ephemerides of orbit determination for the object EGEO07.


Figure 13: Angular distance between observed positions and calculated ephemerides from orbit determination for the object EGEO45.

For the object EGEO45 two short arc orbits of three days were determined, first with the simple DRP model (EPHM_S, no additional parameters, A/M fixed at $0.02 \mathrm{~m}^{2} \mathrm{~kg}^{-1}$, $\mathrm{rms}=8.07$ ") and with the advanced DRP model, an area to mass ratio of $3.0329 \mathrm{~m}^{2} \mathrm{~kg}^{-1}$ was found ( $E P H M-S D R P, \mathrm{rms}=0.86$ "). In addition two long arc orbits of 11 days were determined, first using only the simple DRP model ( $E P H_{M} L$, no additional parameters, A/M fixed at $0.02 \mathrm{~m}^{2} \mathrm{~kg}^{-1}$, $\mathrm{rms}=53.34$ ") and then with the used of the advanced DRP model ( $E P H M \angle L D R P, \mathrm{rms}=0.86$ "), with which an area to mass ratio of 3.0104 $m^{2} \mathrm{~kg}^{-1}$ was estimated.

For the object E06207B it was not possible to determine any orbits with the simple DRP model and the value of 0.02 $m^{2} \mathrm{~kg}^{-1}$ for the area to mass ratio. One orbit was determined with a medium arc length or 15 days, the advanced DRP model was used and an area to mass ratio of $32.3166 \mathrm{~m}^{2} \mathrm{~kg}^{-1}$ was estimated ( $E P H M \_M D R P, \mathrm{rms}=0.54$ "). In addition two long arc orbits of 19 days were determined with the advanced DRP model, in the first case an area to mass ratio of $31.8887 \mathrm{~m}^{2} \mathrm{~kg}^{-1}$ was found ( $E P H M \_L D R P, \mathrm{rms}=5.39$ "), in the second case additional biases were estimated and the area to mass ratio was estimated to be $32.0831 \mathrm{~m}^{2} \mathrm{~kg}^{-1}$ (EPHM_LBIASES, rms=0.95").

Generally it has to be stated that what we call longer arcs for the high area to mass ratio objects are shorter than the ones for GEO or GTO objects with smaller area to mass ratios. This is simply because no good orbits can be determined with longer arcs for high area to mass ratio objects, due to limitations of the DRP model (variable area to mass ratio!). Fig. 11 reveals that the residuals for the orbits with the simple DRP model are the largest independently of the length of the arc that was used. This is not surprising, since the rms of the orbits are remarkably worse and the a priory value for the area to mass ratio is far smaller than the one actually estimated, when the advanced DRP model was used.

For the object EGEO07, which has the smallest area to mass ratio, the ephemerides from the long arc with the advanced DRP model has the least residuals in the long run, but Fig. 12 reveals, that the residuals for the ephemerides from the short arc orbit determination with the advanced DRP model is even better within the first 100 days since orbit determination. Larger variations occur in the intrack component whereas the crosstrack component does not show such variations. Within the first 30 days since orbit determination the angular distance is still around 0.04 degrees within 20 days even below 0.002 degrees. The intrack and crosstrack residuals are for the best ephemerides below 0.7 km and below 0.6 km , respectively, within the first twenty days since orbit determination, but rise to values of 17.1 km and 29.3 km 39 days after orbit determination.


Figure 14: Angular distance between observed positions and calculated ephemerides from orbit determination for the object E06207B.

The situation is different for the object EGEO45, which has a higher area to mass ratio. As shown in Fig.11b large variations in the residuals occur. The smallest residuals are achieved by the short arc orbit determination with the advanced DRP model. For EGE045, which still has an area to mass ratio of below $3.5 \mathrm{~m}^{2} \mathrm{~kg}^{-1}$, the residuals are moderate within the first twenty days since orbit determination, as it can be seen in Fig. 13 and in more detail in Fig.11c, which gives a close look to the first couple of days. The angular distance is below 0.024 degrees, the intrack residuals below 4.28 km and crosstrack below 15.9 km . These residuals are remarkably higher than for the object EGEO07 or for the GEO and high eccentricity objects mentioned in the previous sections, but still in the range of standard TLE ephemerides. Within 30 days since orbit determination the residuals rise up to values of 0.104 degrees in angular distance and 20.5 km and 53.4 km in intrack and crosstrack direction.

Fig.11d shows the residuals for E06207B, which has an area to mass ratio of around $30 \mathrm{~m}^{2} \mathrm{~kg}^{-1}$. Large variations already occur within the first days since orbit determination. The residuals are the smallest for the ephemerides from the longer arc orbit determination with the estimation of biases and the advanced DRP model. But for no epoch a really good solution can be offered. More inside gives Fig.14. It reveals that even four days after the orbit determination, the angular distance is already 0.11 degrees, the intrack and crosstrack residuals are at 33.3 km and 12.6 km .15 days after orbit determination the angular distance is 0.78 degrees, the intrack residuum has a relatively small value of 7.8 km , but the crosstrack residuum is at 283.3 km .

## 5 CONCLUSIONS

Ephemerides generated from TLEs and from different orbit determinations with prior observations were compared with observations.

In case the orbits are determined from prior observations, the smallest residuals for GEO and high eccentricity objects are achieved with observation arcs of about 20 to 30 days. In most of the cases, using the advanced DRP model results in large improvements of the orbit. If the root mean square of the orbit determination becomes smaller when additional biases are estimated the prediction in generally also improves, although the biases are not included in the orbit propagation. In cases where the advanced DRP model does not improve the orbit, the difference in the ephemerides residuals compared to the one of the next better orbit are small. In general the use of the advanced DRP model leads to smaller variations of the residuals from one epoch to the next.

Since the epoch of the last observations used for the orbit determination of the TLEs is unknown, this could not be taken into account in this investigation. In general we conclude, that TLEs show large variations in the angular distance residuals as well as in the intrack and crosstrack residuals.

The expectation values of the absolute values of residuals from TLE ephemerides are in general around 0.02 degrees in angular distance, which result in about 15 km in intrack direction and 7 km in cross track direction. In addition, the standard deviations are of the same order of magnitudes [2]. These values could be confirmed. For GEO objects the residuals of orbit determination ephemerides in angular distance, intrack and crosstrack are below 0.004 degrees, 1.4 km and 2.1 km , respectively, 30 days after orbit determination. For high eccentricity objects the residuals for ephemerides from orbit determination are below 0.006 degrees in angular distance and 2.7 km and 1.77 km in intrack and crosstrack direction, respectively. The quality of the ephemerides from orbit determination is therefore remarkably better in the cases studied here than for ephemerides of TLEs. The residuals are even smaller less than 30 days after orbit determination since last orbit determination. This is crucial when planning follow-up observations, for the correlation of observed tracklets with a catalogue and in the context of collision avoidance.

The situation is different for objects with high area to mass ratios. No official TLEs were available for these objects. Orbits are less good modeled the higher area to mass values are reached. The ephemerides from orbit determinations with shorter arcs have smaller residuals than good orbits from longer arcs. Examples were shown, that for moderate values of the area to mass rations the residuals after 20 days are still below 0.025 degrees in angular distance and below 4.3 km and 16.0 km in intrack and crosstrack direction, respectively. For an area to mass ratio of over 30 even within the first days after orbit determination the angular distance is already 0.1 degrees and the intrack and crosstrack residuals are in the order of 30 km and 15 km , respectively. Generally orbit determination is more demanding for objects with higher area to mass ratios. Consequently also the quality of the predicted ephemerides is lower for objects with higher area to mass ratio objects.

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## REFERENCES

[1] D. Vallado and W. McCain. Fundmentals of Astrodynamics and Applications. Microcosm Press, El Sugundo, California, 2001. ISBN 0-7923-6903-3.
[2] C. Früh, T. Schildknecht, R. Musci, and M. Ploner. Catalogue Correlation of Space Debris Objects. In Proceedings of the Fifth European Conference on Space Debris, ESOC, Darmstadt, Germany, 30 March-2 April 2009, 2009.
[3] G. Beutler. Methods of Celestial Mechanics. two volumes. Springer-Verlag, Heidelberg, 2005. ISBN: 3-540-40749-9 and 3-540-40750-1.
[4] T.A. Springer. Modelling and Vallidating Orbits and Clocks Using the Global Positioning System, volume 60 of Geodätisch-geophysikalische Arbeiten in der Schweiz. Schweizerische Geodätische Kommission, 2000.
[5] F.R. Hoots, R.L. Roehrich, and T.W. Kelso. Models for Propagation of NORAD Element Sets. Spacetrack Report, No. 3, 1980.
[6] R. Musci. Analyzing long Observation Arcs for Objects with high Area-to-Mass Ratios in Geostationary Orbits. In Proceedings of the International Astronautical Congress 2008, A6.1.10, Glasgow, Scotland, Great Britain, 29 Sep- 3 Oct, 2008, 2008.

