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OBSERVATIONS OF HIGH ALTITUDE OBJECTS FROM MULTIPLE SITES

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Abstract

The Astronomical Institute of the University of Bern (AIUB) was and is conducting several search campaigns for space debris in the geostationary ring (GEO) and the geostationary transfer orbit (GTO). First tests and studies to build up a catalogue of satellites and space debris were performed in the previous years. The studies included only observations from one site. The benefit of using several sites is studied in this article.

The main task of building up a catalogue is the acquisition of a "secured" orbit, i.e., of an orbit allowing a safe recovery of the object after several weeks to months after the epoch of the last observation used for the orbit determination. Observations from additional sites may help to reduce the number of follow-up observations that are needed to determine a "secured" orbit. The formal errors of the orbital elements resulting from the orbit determination using simultaneous observations from two and three sites are analyzed. In order to study the influence of the observation geometry, the longitude and latitude differences between the sites were varied.

Truly simultaneous observations are difficult to realize. Additional sites can, however, also be used to perform the first follow-up tracks after a short time interval. This time interval between the initial track and the follow-up track was systematically varied and the dependency of the accuracy of the orbits on the time interval analyzed. The formal errors of the orbital elements resulting from simulations for two sites were compared with those resulting from simulations for only one site with the same time interval between the tracks. Our studies show that the formal errors of the obtained orbital elements also depend on the position of the object with respect to the observing sites.

Introduction

Various institutes have performed search surveys for space debris during the last years. The Astronomical Institute of the University of Bern (AIUB) is conducting optical search campaigns for high altitude objects using the ESA Space Debris Telescope (ESASDT) on Tenerife on behalf of ESA. The aim of these campaigns is to improve the statistical information about the populations of objects in geostationary orbits (GEO) and geostationary transfer orbits (GTO) and to perform first tests to build up a catalogue of debris objects. A large amount of faint and unknown objects have been observed within these surveys ([1]). Only a short observation arc is available for most of these objects.

These short arcs do not allow determining an accurate full six parameter orbit. Normally, circular orbits are determined instead. A circular orbit is a good approximation for GEO, but not for eccentric orbits like GTO.

Joint observations of GEO objects have been performed since June 2004 by several astronomical observatories ([2]). The program is lead by the Russian Academy of Science (RAS). AIUB is contributing to this program using its own 1 meter telescope in Zimmerwald (ZIMLAT). The aim of the program is to continuously track recently discovered unknown objects over a longer time frame.

A possible concept for a catalogue of objects in GEO was developed by [3] in the framework of ESA studies for a European Space Surveillance Network ([4] and [5]). AIUB participated in these studies, whereas the work focused on the selection of optical detectors, the development of survey strategies for high-altitude orbits, and on the performance estimation of the GEO part ([6]).

A test campaign for acquiring simultaneous optical observations from two sites was performed in [7] but with poor results due to the bad weather conditions. The test illustrates the main problem of simultaneous observations from multiple sites: the weather conditions have to be good for at least two of the involved sites.

The aim of this work is to study the benefit of observations from multiple sites compared to observations from one site using simulations. Only GEO objects are considered.

Observations from one Site

The discovery track of an object usually consists of a small number (two to ten) of observations. The track length is only a few minutes. Such a short arc does normally not allow determining all six orbital elements. Follow-up observations are therefore needed in order to get a longer observation arc. Follow-ups from several nights are needed if the orbit should be accurate enough to be included into a catalogue.

[8] showed that at least two follow-up tracks are needed to recover a newly discovered GEO object during the following night. The ideal time interval between the tracks was found to be one hour. This on the one hand allows recovering the object with a small field of view (FOV) of $0.4°$ and on the other hand assures that the required observation arc length from the night of the discovery will be short, which allows recovering an object discovered late in the observation night.

Another follow-up track has to be observed during the following night to determine a "secured" orbit, i.e., an orbit, which guarantees a save recovery of the object after a few weeks. Such an orbit may be used for a catalogue.

Observations from Multiple Sites

Instead of improving the orbit by observing the object at different epochs, the orbit can also be improved by observing the object from different sites. Simultaneous observations from multiple sites allow determining the distance of the object with a higher accuracy than from a short arc of observations from a single site due to the improved geometry. Generally, the accuracy of the orbit depends on the geometry of the observing sites during the observations. For two sites, the geometry is characterized by the differences $\Delta\lambda$ in longitude λ, $\Delta\beta$ in latitude β, and the longitude λ_0 of the object. As we will see later, a larger separation between the sites reduces the error of the estimated orbit.

The orbit improvement resulting from observations from two and three sites are studied in the following. Observations from hypothetical sites were simulated for this purpose. The main site was assumed to be located at longitude $\lambda = 0°$ and latitude $\beta = 0°$. For four other sites, the longitude was fixed to $\lambda = 0°$ and the latitudes to $10°$, $20°$, $30°$, and $40°$. The latitude was fixed to $\beta = 0°$ and the longitude varied from 10° to 40° for another four sites.

Simulated Orbits

The same 250 simulated GEO orbits (from now on called "true" orbits) as in the work [3] were used. The elements were randomly varied within the ranges listed in Table 1. The longitude range was selected in such a way that the objects are visible from European sensors at the initial epoch. The right ascension of the ascending node Ω of the majority of uncontrolled GEO objects is strongly correlated with the inclination. For inclinations $i > 0.5°$ the right ascension of the ascending node Ω was therefore approximated by the function ([9]):

$$\cos\Omega = \cot 7.5° \, \frac{1-\cos i}{\sin i}. \quad (1)$$

Each simulated observation track within this work consists of three observations separated by 30 seconds. An rms error of $\sigma = 0.5''$ was assumed for the accuracy of the single observation. Furthermore, a transparent Earth is assumed, i.e., the objects can be observed wherever their position is on the orbit.

The perturbations due to the Earth's oblateness, the lunar and solar gravity were included in all performed simulations and orbit determinations.

Simultaneous Observations

Let us first study the orbit accuracy for simultaneous observations from two sites. For comparison, circular orbits were determined from single tracks observed from the main site only. The predicted differences Δ with respect to the true orbit for this case are shown in Figure 1. The differences were determined with

$$\Delta = \arccos(\sin\delta_t \, \sin\delta_d + \cos\delta_t \, \cos\delta_d \, \cos\Delta\alpha), \quad (2)$$

where

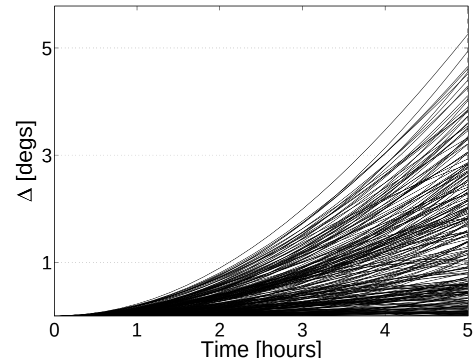

Figure 1: Difference Δ between "true" and determined circular orbit of a GEO object representing one track observed from a virtual site located at $\lambda = 0°$ and $\beta = 0°$.

δ_t is the declination from the "true" orbit,

δ_d is the declination from the determined orbit,

$\Delta\alpha$ is the difference between the right ascension α of the 'true" and the determined orbit.

For almost all objects, the differences are below 5° after 5 hours.

One observation track was simulated for each of the other eight sites using the same epoch as for the main site. Orbits were determined using the observations from the main site together with one of the tracks from the other sites. Although the observation arc is very short, the geometry allows determining all six orbital elements.

Let us first look at the results achieved with the sites at the same latitude as the main site, but at different longitudes in Figure 2. Note that the scale in the y-axis is different in the four subfigures. Some objects with large differences ("outliers") show up in each of the four subfigures. They are the smallest for the site with $\lambda = 40°$. An explanation for the outliers will be given later. Nevertheless, it can also be clearly seen that for a majority of the objects the differences are getting smaller for a larger separation between the sites. For a difference between the sites of 40° in longitude most of the differences are below 1° after 5 hours. If we disregard the outliers the results are by a factor of 5 better

Table 1: Range of the orbital elements used for the simulation of 250 GEO orbits.

Semi-major axis	$40164\,\text{km} < a < 44164\,\text{km}$
Eccentricity	$0.00 < e < 0.05$
Inclination	$0^\circ < i < 15^\circ$
R.A.of ascending node	$0^\circ < \Omega < 360^\circ$
Argument of perigee	$0^\circ < \omega < 360^\circ$
Longitude at t_0	$-70^\circ < \lambda < 120^\circ$

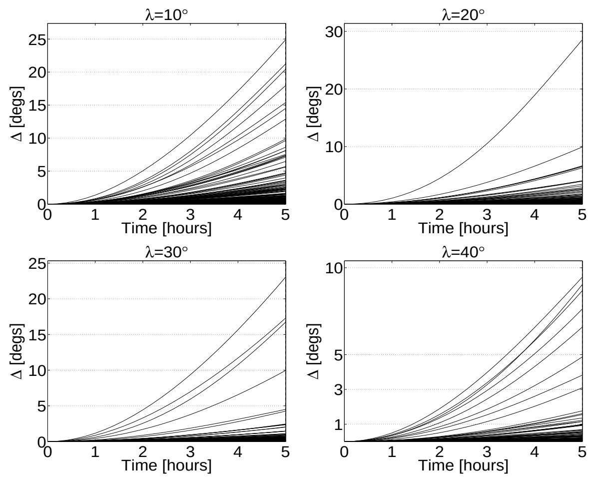

Figure 2: Difference Δ between “true” and determined elliptical orbit of a GEO object representing two tracks observed simultaneously from two virtual sites with identical latitude and different longitudes.

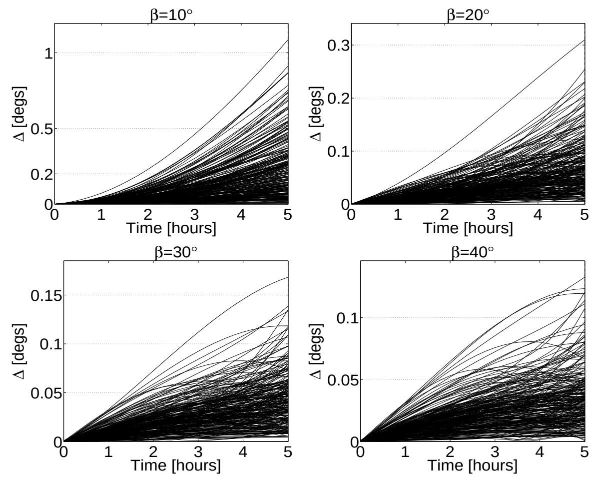

Figure 3: Difference Δ between “true” and determined elliptical orbit of a GEO object representing two tracks observed simultaneously from two virtual sites with identical longitude and different latitudes.

than those achieved with observations from the main site only.

The averages of the formal errors (from now on called “mean formal errors”) for the elements resulting from the orbit determination are given in Table 2. The accuracy for all elements improves with the separation in longitude between the sites. The mean formal errors are, however, affected by the outliers.

Much better results are achieved when using sites with identical longitudes but different latitudes. In Figure 3, no outliers comparable to those in Figure 2 are seen. Already for a site separation of 10° in latitude, the differences are smaller than 1° after 5 hours. The results for latitude separations larger than 30° are by a factor of about 50 better than those based on one site only.

The mean formal errors for the sites with identical longitude and different latitudes in Table 3 are mostly smaller than in Table 2, except for i and Ω. The errors of a and e are by a factor of 3 smaller. Also, the accuracies of ω and T_0 are slightly better, but the improvement is not of the same magnitude.

Using simultaneous observations from two sites mainly allows for better determination of the shape of the orbit, while the improvement in the orbital plane is rather small. The best result was achieved with a large site separation in latitude. But all results are much better than those for an orbit determined from one track observed from one site only. Nevertheless, additional observations during the night of the dis-

Table 2: Mean formal errors for the elliptical orbit determination representing two tracks of a GEO object observed simultaneously from two virtual sites with identical latitude and different longitudes.

λ [°]	a [m]	e	i [°]	Ω [°]	ω [°]	T_0 [s]
10	$3.82{\cdot}10^5$	$2.94{\cdot}10^{-2}$	$2.27{\cdot}10^{-2}$	0.26	72.3	$1.77{\cdot}10^4$
20	$2.23{\cdot}10^5$	$1.52{\cdot}10^{-2}$	$1.54{\cdot}10^{-2}$	0.21	54.9	$1.31{\cdot}10^4$
30	$1.82{\cdot}10^5$	$1.23{\cdot}10^{-2}$	$1.35{\cdot}10^{-2}$	0.19	40.3	$9.66{\cdot}10^3$
40	$1.52{\cdot}10^5$	$1.03{\cdot}10^{-2}$	$1.27{\cdot}10^{-2}$	0.19	38.2	$9.27{\cdot}10^3$

Table 3: Mean formal errors for the elliptical orbit determination representing two tracks of a GEO object observed simultaneously from two virtual sites with identical longitude and different latitudes.

β [°]	a [m]	e	i [°]	Ω [°]	ω [°]	T_0 [s]
10	$1.59{\cdot}10^5$	$1.17{\cdot}10^{-2}$	$1.58{\cdot}10^{-2}$	0.27	54.3	$1.31{\cdot}10^4$
20	$8.20{\cdot}10^4$	$5.93{\cdot}10^{-3}$	$1.34{\cdot}10^{-2}$	0.26	36.2	$8.74{\cdot}10^3$
30	$5.67{\cdot}10^4$	$4.08{\cdot}10^{-3}$	$1.29{\cdot}10^{-2}$	0.26	26.1	$6.30{\cdot}10^3$
40	$4.46{\cdot}10^4$	$3.20{\cdot}10^{-3}$	$1.28{\cdot}10^{-2}$	0.25	20.5	$4.92{\cdot}10^3$

covery are needed to recover all objects during the following night.

It can be expected that the results become even better when three sites are observing an object simultaneously. From the 16 possible combinations of the sites, only those 4 where the sites have the same separation in longitude and latitude from the main site were analyzed.

Table 4 summarizes the resulting mean formal errors. The errors from two sites separated by $\Delta\beta$ =40° are included in the last row for comparison. The errors for three sites and separations of 40° in longitude and latitude are about 30% smaller that for two sites separated by 40° in latitude. Whereas the mean formal errors of a, e, ω, and T_0 for $\beta = 40°$ are smaller than those for $\lambda, \beta = 20°$, the orbital plane is in all cases better determined with observations from three sites.

Improvements resulting from simultaneous observations of an object from three sites are marginal compared to simultaneous observations from two sites separated by several ten degrees. It is therefore not recommended to use more than two telescopes simultaneously for the acquisition of "secured" orbits. A third telescope could be better used for other tasks, e.g., follow-up observations of detected objects or performing a survey of another region.

Follow-up After One Hour

The task to discover an object simultaneously from two or more sites might be difficult to be realized in practice. Alternatively one may perform the first follow-up observations from a second site. The first follow-up tracks were simulated one hour after the discovery for all sites. For comparison, follow-up tracks were also simulated for the main site. The differences Δ between the true and the estimated elliptical orbits for this case are shown in Figure 4. After 5 hours, the differences are still smaller than 3° for all objects.

Elliptical orbits were determined using the first track from the main site together with a follow-up track after one hour observed from one of the other sites. Some outliers still show up for the resulting orbits including follow-up observations from the sites with different longitude (Figure 5). But they are less prominent than in Figure 2. In general, the differences seem to be smaller than for simultaneous observations.

Table 4: Mean formal errors for the elliptical orbit determination representing three tracks of a GEO object observed simultaneously from three virtual sites with different longitudes and latitudes.

$\lambda, \beta\,[°]$	a [m]	e	$i\,[°]$	$\Omega\,[°]$	$\omega\,[°]$	T_0 [s]
10	$1.05{\cdot}10^5$	$7.39{\cdot}10^{-3}$	$9.76{\cdot}10^{-3}$	0.16	36.2	$8.71{\cdot}10^3$
20	$5.54{\cdot}10^4$	$3.79{\cdot}10^{-3}$	$8.11{\cdot}10^{-3}$	0.15	37.0	$9.04{\cdot}10^3$
30	$3.93{\cdot}10^4$	$2.67{\cdot}10^{-3}$	$7.84{\cdot}10^{-3}$	0.15	16.6	$4.02{\cdot}10^3$
40	$3.17{\cdot}10^4$	$2.14{\cdot}10^{-3}$	$7.88{\cdot}10^{-3}$	0.16	13.4	$3.23{\cdot}10^3$
$\beta = 40°$	$4.46{\cdot}10^4$	$3.20{\cdot}10^{-3}$	$1.28{\cdot}10^{-2}$	0.25	20.5	$4.92{\cdot}10^3$

Table 5: Mean formal errors for the elliptical orbit determination representing the discovery track of a GEO object observed from the main site and the follow-up after 1 h. The follow-up observations were simulated for virtual sites with identical latitude and different longitudes.

$\lambda\,[°]$	a [m]	e	$i\,[°]$	$\Omega\,[°]$	$\omega\,[°]$	T_0 [s]
0	$1.64{\cdot}10^6$	$2.28{\cdot}10^{-2}$	$8.39{\cdot}10^{-3}$	0.17	43.8	$9.96{\cdot}10^3$
10	$1.39{\cdot}10^6$	$2.00{\cdot}10^{-2}$	$5.16{\cdot}10^{-3}$	0.12	37.2	$8.56{\cdot}10^3$
20	$7.12{\cdot}10^5$	$9.88{\cdot}10^{-3}$	$3.66{\cdot}10^{-3}$	$9.96{\cdot}10^{-2}$	23.4	$5.33{\cdot}10^3$
30	$5.21{\cdot}10^5$	$7.09{\cdot}10^{-3}$	$3.25{\cdot}10^{-3}$	$9.32{\cdot}10^{-2}$	21.8	$5.02{\cdot}10^3$
40	$3.68{\cdot}10^5$	$4.66{\cdot}10^{-3}$	$3.05{\cdot}10^{-3}$	$8.23{\cdot}10^{-2}$	18.4	$4.23{\cdot}10^3$

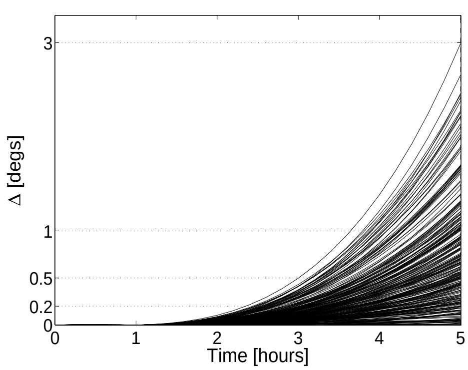

Figure 4: Difference Δ between "true" and determined elliptical orbit of a GEO object representing two tracks separated by 1 h observed from a virtual site located at $\lambda = 0°$ and $\beta = 0°$.

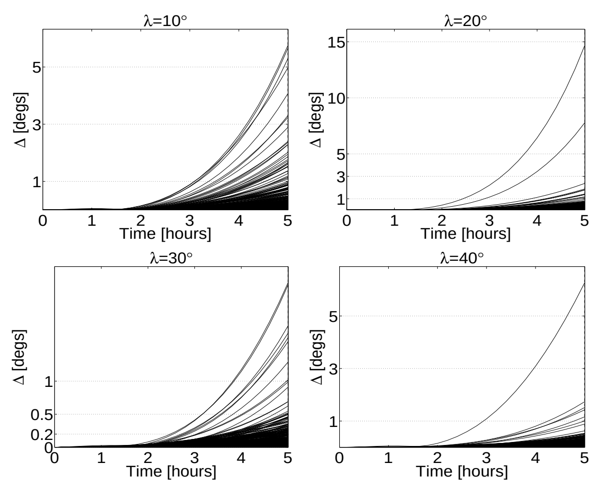

Figure 5: Difference Δ between "true" and determined elliptical orbit of a GEO object representing one track observed from the main site and one follow-up track after 1 h observed from virtual sites with identical latitude and different longitudes.

Table 5 summarizes the mean formal errors. We see that the orbital planes are better determined than for simultaneous observations (Table 2). Except for the semi-major axis, the errors are smaller for the follow-ups after 1 hour comparing the same separation between the sites. The inclination is by a factor of 3 better determined. As we have seen in Figure 5, the larger error in the semi-major axis is compensated by the improvement of the other elements, resulting in smaller differences than for simultaneous observations.

Again, the results are different for the sites with identical longitude and different latitudes. The

Table 6: Mean formal errors for the elliptical orbit determination representing the discovery track of a GEO object observed from the main site and the follow-up after 1 h. The follow-up observations were simulated for virtual sites with identical longitude and different latitudes.

$\beta\,[^\circ]$	$a\,[\mathrm{m}]$	e	$i\,[^\circ]$	$\Omega\,[^\circ]$	$\omega\,[^\circ]$	$T_0\,[\mathrm{s}]$
0	$1.64{\cdot}10^6$	$2.28{\cdot}10^{-2}$	$8.39{\cdot}10^{-3}$	0.17	43.8	$9.96{\cdot}10^3$
10	$5.22{\cdot}10^5$	$6.60{\cdot}10^{-3}$	$1.24{\cdot}10^{-2}$	0.86	21.5	$4.80{\cdot}10^3$
20	$2.73{\cdot}10^5$	$3.46{\cdot}10^{-3}$	$1.26{\cdot}10^{-2}$	0.49	15.4	$3.53{\cdot}10^3$
30	$1.86{\cdot}10^5$	$2.47{\cdot}10^{-3}$	$1.26{\cdot}10^{-2}$	0.42	11.6	$2.89{\cdot}10^3$
40	$1.43{\cdot}10^5$	$2.01{\cdot}10^{-3}$	$1.25{\cdot}10^{-2}$	0.38	9.7	$2.25{\cdot}10^3$

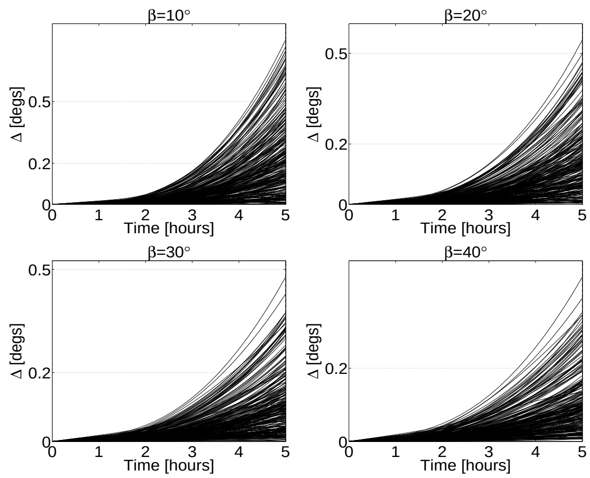

Figure 6: Difference Δ between "true" and determined elliptical orbit of a GEO object representing one track observed from the main site and one follow-up track after 1 h observed from virtual sites with identical longitude and different latitudes.

differences shown in Figure 6 are smaller than those for the sites with different longitudes, and no outliers occur. But the difference between the two cases is not as large as in the case of simultaneous observations. Interestingly, the differences are larger for an observation arc of one hour than for the simultaneous observations when comparing only the results for the sites at different latitudes. This is an indication that in this case the observation geometry is better for the simultaneous observations.

Table 6 shows that the orbital plane is not as well determined when using the observations from two sites at different latitudes as for observations from one site (first row). Furthermore, the plane is not as well determined as for the sites with identical latitude and different longitudes. The other elements, however, are by a factor of about two better determined. Taking into account the results from Figures 5 and 6 we conclude that the difference between the "true" and the estimated orbit is dominated by the accuracy of the orbital shape (a and e) rather than by the accuracy of the orbital plane. Comparing the mean formal errors for an observation arc of one hour with those for simultaneous observations (Table 3) we see that the errors for Ω are larger, but the error for i are almost identical. I.e., the orbital plane is slightly less accurately determined. The errors for the elements e, ω, and T_0, on the other hand, are slightly smaller. The larger differences in Figure 6 compared to Figure 3 are mainly explained by the larger errors in the semi-major axes.

Development of the Formal Errors

The previous section showed that the orbit accuracy does not necessarily improve with the length of the observation arc. It does improve if the observations are performed from one site. When using the observations from two sites, the accuracy of the orbits can be deteriorated if the observation tracks are separated by one hour compared to simultaneous observations.

The development of the mean formal errors with longer gaps between the observation tracks is studied in the following. For this purpose, the main site with $\lambda = 0°$, $\beta = 0°$, one site with a large separation in longitude ($\lambda = 40°$, $\beta = 0°$), and one with a large separation in latitude ($\lambda = 0°$, $\beta = 40°$) were selected. The first observation tracks were in all cases assumed to be acquired from the main site. A second observation track was simulated for each of the three sites after different time gaps. Elliptical orbits were determined from the two tracks.

The mean formal errors σ for the six orbital elements as a function of the the time interval Δt between the two tracks are shown in Figure 7. To avoid misleading structures due to outliers the mean formal errors were calculated without the largest 5% and the smallest 5% of the formal errors. The errors for the case where both tracks were observed from the main site are marked with circles connected with a solid line. They are from now on called "Δ-errors". There are no data points at $\Delta t = 0$ hours as no reasonable elliptical orbit can be determined from such a short observation arc observed from one site. Those mean formal errors, where the second track was observed from the site with a large separation in latitude, are marked with $\times$ connected by a dotted line ("Δ_β-errors"). The + symbols connected with a dashed line mark the errors for the sites at different longitudes ("Δ_λ-errors"). Note that the scale of the y-axis is logarithmic.

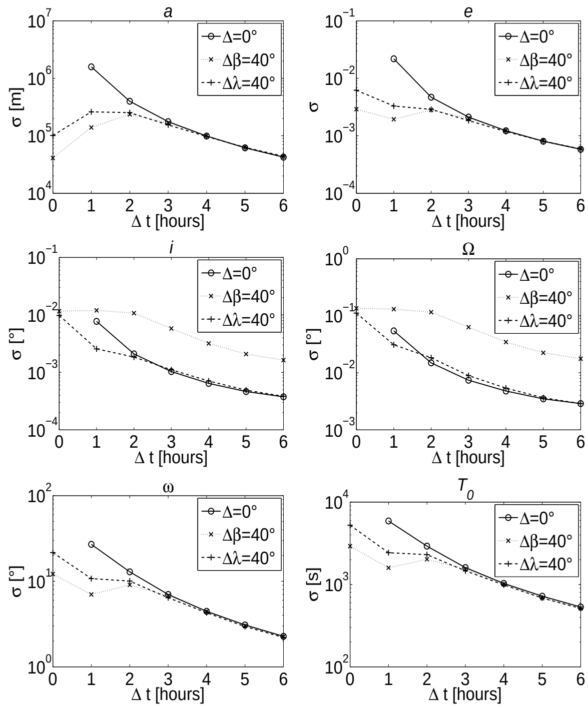

Figure 7: Mean formal errors σ for the elliptical orbit determination representing two tracks of a GEO object separated by Δt observed from one virtual site ($\circ$) or two virtual sites with identical longitude and different latitude ($\times$) and identical latitude and different longitude (+). The scale of the y-axis is logarithmic.

Figure 7 (top, left) shows the errors for the semi-major axis a. As expected, the Δ-errors decrease with the length of the observation arc. The Δ_β-errors and the Δ_λ-errors get larger at the beginning and then improve after 2 hours and 1 hour respectively. Both converge to the Δ-errors after a few hours. The errors are almost identical for observation arcs longer than four hours. Note that the Δ_β-errors for $\Delta t = 0$ hours are almost as small as the Δ-errors for an arc of six hours!

The errors of the eccentricity e show a similar development as the errors of the semi-major axis a. The main difference is, that the errors for simultaneous observations are larger than for an observation arc of one hour. The Δ_β-errors for $\Delta t = 0$ hours are of the same magnitude as the Δ-errors for an arc of three hours.

The accuracy of the orbital plane is given by Figure 7 (mid, left) and (mid, right). The Δ-errors for the inclination i improve monotonically with Δt. The Δ_λ-errors are clearly smaller for an arc of one hour and slightly smaller for an arc of two hours compared to the Δ-errors. From there onwards, the accuracy is better for the Δ-errors. The Δ_β-errors are in all cases clearly larger than the others. The errors in the right ascension of the ascending node Ω resembles the figure for i. The Δ-errors, however, are already smaller than the Δ_λ-errors after two hours.

The errors for ω and T_0 are similar to those of the eccentricity. The Δ-errors, however, are slightly larger than the others for an arc longer than three hours.

The above comparison showed that an arc of about $3-4$ hours of observations from a single site is needed to get a comparable accuracy as in the case of simultaneous observations made from two sites separated by 40° in latitude. A separation in longitude, which is normally realized in a space surveillance system, shows about the same quality. We therefore may conclude that two instruments located at two different sites should be used to perform simultaneous observations in order to acquire a "secured" orbit. A shorter observation arc would then be needed to be able to recover the object during the following night. But the loss of observation time has also to be considered as the second instrument cannot be used for other tasks. Another restriction is given by the fact that the available observation time per night is shorter as the object has to be visible from both sites during the night. Three sites uniformly distributed in longitude would then not be sufficient to cover the whole GEO belt.

In general, the errors related to one and two sites, respectively, converge for longer observation arcs, implying that the selection of the site has no important influence on the accuracy of the orbit for an arc length of several hours. In this case the site with the best observation conditions, namely the best phase angle, should be selected if the objects is visible from more than one site.

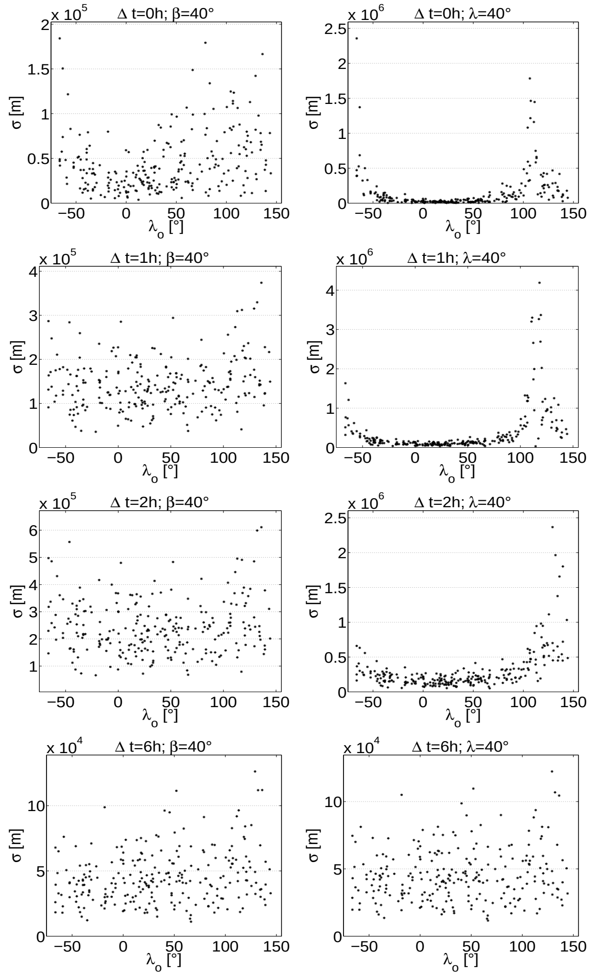

Figure 8: Mean formal errors σ for the semi-major axis of the elliptical orbit determination representing two tracks of a GEO object separated by 0 h, 1 h, 2 h, and 6 h observed from two virtual sites with identical longitude and different latitude (left) and identical latitude and different longitude (right) plotted against the longitude λ_o of the objects.

Dependency on the Object Position

We have seen that the orbits are better determined when using the observation from two sites located at different latitudes than at different longitudes. This finding is not obvious, as the geocentric angles between the sites are the same. The angle between the two sites as seen from the object is, however, in general different for the two cases. This angle roughly has the same size for objects located in longitudes and latitudes between the considered stations. GEO objects are distributed within a narrow band between +17° and −17° latitude. The average angle under which two sites are seen from GEO objects is therefore different for two sites on the equator and for sites at different latitudes.

To demonstrate the impact of the object position on the orbital accuracy, the mean formal errors σ of the semi-major axis are shown as a function of the longitude λ_o of the object (Figure 8). The left column of figures represent the Δ_β-errors, the right column the Δ_λ-errors. The length of the observation arc is the same for figures in the same row. The observations for the top row of figures were assumed to be acquired simultaneously from the two sites. The rows below are for observation arcs of 1 hour, 2 hours, and 6 hours.

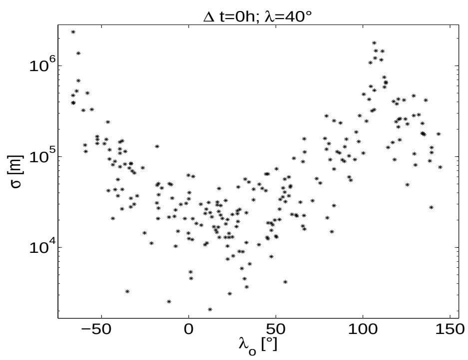

Figure 9: Mean formal errors σ for the semi-major axis of the elliptical orbit determination representing two tracks of a GEO object observed at the same epoch from two virtual sites with identical latitude $\beta = 0°$ and a separation in longitude of $\Delta\lambda = 40°$. The scale of the y-axis is logarithmic.

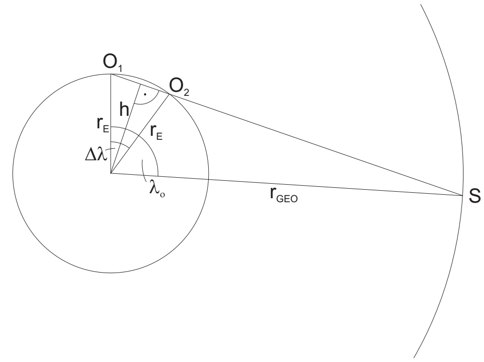

Figure 10: Longitude λ_o of a GEO satellite S that is seen in the same direction from two observers O_1 and O_2.

Let us first look at the Δ_β-errors. In the top figure, a minimum is found close to $\lambda_o = 0°$. The errors in this range are mostly below $\sigma = 50\,000$ m. The largest errors are around $\lambda_o = -70°$, $\lambda_o = 70°$, and $\lambda_o = 140°$, where they reach almost 200 000 m. No clear minimum can be found in the three figures below the top figure. The distribution of the errors is uniform.

The top three figures for the Δ_λ-errors are different from those for the Δ_β-errors. Two strong peaks are at $\lambda_o > 100°$ and at $\lambda_o < -50°$. In the figure for $\Delta t = 0$ hours, the peaks are located at $\lambda_o = -66.3°$ and $\lambda_o = 106.5°$. The errors are rather symmetrically distributed around $\lambda_o = 20°$. Objects with $\lambda_o = 20°$ are located exactly in the middle between the two sites. The observation geometry is obviously ideal for these objects. Therefore, we have to expect the smallest errors at $\lambda_o = 20°$. Figure 9 shows that this is really the case. The same data points as in Figure 8 (top, right) are shown, but with a logarithmic scale for the y-axis. The minimum actually is close to $\lambda_o = 20°$.

An analysis not described in this paper has shown that the distance between the peaks depends on the separation between the sites. It is larger for a larger separation in longitude between the sites. This is explained by Figure 10, which shows the Earth and part of a GEO as seen from the North pole. It is assumed that the two sites O_1 and O_2 as well as the orbit of the GEO object lie in the equator plane. The peaks result when an object is observed in the same direction from O_1 and O_2 (remember that a transparent Earth is assumed). The longitude λ_o of such an object can then be determined with

$$\lambda_o = \frac{\Delta\lambda}{2} \pm \arccos\left(\frac{h}{r_{GEO}}\right), \qquad (3)$$

$$\text{with } h = r_E \cdot \cos\frac{\Delta\lambda}{2}. \qquad (4)$$

r_e and r_{GEO} are the radii of the Earth and of the GEO object, respectively. For a separation of $\Delta\lambda = 40°$ between the sites the resulting longitude is $\lambda_o = 20° \pm 81.8°$, i.e., we expect the peaks at $-61.8°$ and at $101.8°$. This finding is confirmed by Figure 8 (top, right).

In Figure 8 the distance between the peaks is greater for $\Delta t = 1$ h and $\Delta t = 2$ h. Furthermore, the errors around $\lambda_o = 20°$ are getting larger. For an object with a longitude between or close to the longitudes of the two sites the orbit is therefore better determined with simultaneous observations than with observations separated by a few hours.

The peaks in these figures also explain the outliers in Figure 2 and 5. Larger formal errors result in larger differences between the "true" and the determined orbit.

The two figures at the bottom are very similar. Both figures show neither a peak nor a minimum. The accuracies of these orbits are rather dominated by the length of the observation arc than by the location of the sites.

Examples Using Real Observations

A large amount of observations has been gathered by the ESASDT, the ZIMLAT, and the joint observations program led by RAS. These observations were searched for objects observed from two sites during the same night. In addition, some of the observations had to be acquired within half an hour from both sites and a few hours before that epoch from one of the two sites. Unfortunately, these requirements could not be met frequently. Nevertheless, some examples can be analyzed here. The examples include observations from the ESASDT, the ZIMLAT, and the Crimean Astrophysical Observatory (CRAO) in Nauchny.

The separation between two sites has an impact on the accuracy of the determined orbit. The separations in longitude and latitude between the three sites of Nauchny (CRAO), Zimmerwald (ZIMLAT), and Tenerife (ESASDT) are given in Table 7. The separations are given form East to West in longitude and from North to South in latitude. The sites of ZIMLAT and ESASDT have about the same separation in longitude and in latitude. Such a constellation was not used in the previous simulations. The two sites CRAO and ZIMLAT, however, are a good example as they are located nearly at the same latitude. For the sites CRAO and ESASDT the separation in longitude is much larger than in latitude.

A first example of a GEO object (GEO_M1) meeting the requirements is shown in Table 8, which gives the formal errors resulting from the determination of elliptical orbits using two observation tracks. The time interval Δt between the two tracks is given in the first column. The next six columns contain the formal errors of the orbital elements. The RMS resulting from the orbit determination is given in the last column.

Table 7: Separations in longitude λ and latitude β between the three sites of Nauchny (CRAO), Zimmerwald (ZIMLAT), and Tenerife (ESASDT).

	$\Delta\lambda$ [°]	$\Delta\beta$ [°]
ZIMLAT - ESASDT	23.58	20.35
CRAO - ZIMLAT	26.73	-2.09
CRAO - ESASDT	50.31	18.26

The formal errors for the orbits determined from observations from CRAO only are shown in the upper part of the table, whereas the lower part shows the formal errors from observations from CRAO and ZIMLAT. In each case, the first observation track was observed with CRAO, while the second track was observed either from CRAO or from ZIMLAT.

Figure 7 let us expect that the formal error of the semi-major axis is larger for a gap of one hour between the tracks than for almost simultaneous observations. This is exactly what resulted for the object GEO_M1. It was unexpected, however, that the difference between the formal errors from one site compared to two sites is so small. But we have to consider that the separation between the two sites was 40° for Figure 7 and only about 25° for CRAO and ZIMLAT.

The formal errors for i and Ω are slightly smaller for the longer gap. In addition, the formal errors for one site are larger than for two sites. This is more or less what was expected.

For e, ω, and T_0 the formal errors are larger for the longer gap. From Figure 7 we expect smaller formal errors for all three cases. Why does the example based on real observations not match the results from the simulations? In Figure 7 we gave the mean formal errors, i.e., we cannot see whether the errors of all objects show the same development or not. Therefore, the formal errors for $\Delta\lambda = 40°$ are provided in Figure 11 for all 250 simulated objects. The left figure shows the formal errors of the semi-major axis, while the right figure shows those of the eccentricity. It can be seen that the main structure is correctly represented by the

Table 8: Formal errors for the elliptical orbit determination of object GEO_M1 representing two tracks observed with the CRAO (top) and two tracks observed with the CRAO and the ZIMLAT (bottom).

CRAO

Δt [h]	a [m]	e	i [°]	Ω [°]	ω [°]	T_0 [s]	RMS [″]
1.21	$3.14 \cdot 10^5$	$5.49 \cdot 10^{-3}$	$8.88 \cdot 10^{-3}$	0.32	17.66	$4.26 \cdot 10^3$	0.46

CRAO and ZIMLAT

Δt [h]	a [m]	e	i [°]	Ω [°]	ω [°]	T_0 [s]	RMS [″]
0.26	$4.75 \cdot 10^4$	$1.89 \cdot 10^{-3}$	$1.02 \cdot 10^{-2}$	0.22	30.79	$7.26 \cdot 10^3$	0.30
1.21	$2.38 \cdot 10^5$	$4.06 \cdot 10^{-3}$	$7.52 \cdot 10^{-3}$	0.20	67.55	$1.57 \cdot 10^4$	0.41

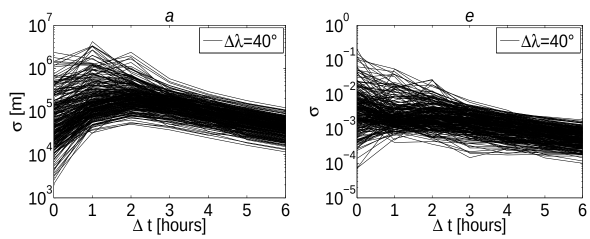

Figure 11: Formal errors σ of the semi-major axes (left) and the eccentricity (right) for the elliptical orbit determination representing two tracks of 250 GEO objects separated by Δt observed from two virtual sites with identical latitude and a separation of 40° in longitude. The scale of the y-axis is logarithmic.

mean formal errors in Figure 7. But Figure 11 also shows that the development may differ very much from case to case for $\Delta t < 3$ hours. The formal errors of the eccentricity of GEO_M1 are therefore consistent with the simulations. For some simulated objects, the formal errors are also larger for a gap of one hour than for simultaneous observations. The figures for the formal errors of ω and T_0 for the 250 simulated objects are not shown as they are similar to the one for e.

A second example of a real object (GEO_M2) is given in Table 9. The upper part shows the formal errors resulting from the observations from CRAO. Observation tracks observed from CRAO and from ESASDT were used to determine the formal errors in the lower part. The time interval between the almost simultaneously observed tracks is shorter than in the previous example, whereas the longer gap is almost three hours.

For a gap of three hours, we would expect that the formal errors for the orbits determined from observations from one site and those from two sites have nearly the same value. For object GEO_M2, the formal errors for one site are clearly smaller than for two sites, exept for those of ω and T_0. The RMS is smaller, as well. As the formal errors are proportional to the RMS, the value of the RMS has to be considered, too. This example is therefore consistent with the simulations, as well.

Object GEO_M3 (Table 10) provides another example. For this object, there was not only one follow-up track after about one hour available from two sites, but also after almost eight hours. The object was observed by the ZIMLAT and the ESASDT.

For this object, the formal errors of a, e, and i show the expected behaviour seen in the simultaneous observations to a gap of one hour. This is not the case for the other three elements. But as we have indicated with Figure 11 this is not unexpected. A clear improvement of the accuracy for a gap of eight hours can be seen for all elements. When comparing the formal errors achieved from two sites to those from one site, we have to consider the RMS values. The RMS is much smaller for the orbits determined from the observations from two sites than from one site. This also reflects in the formal errors for a gap of one hour and eight hours, which are smaller than we would expect from Figure 7. The formal errors from two sites are by fac-

Table 9: Formal errors for the elliptical orbit determination of object GEO_M2 representing two tracks observed with the CRAO (top) and two tracks observed with the CRAO and the ESASDT (bottom).

CRAO

Δt [h]	a [m]	e	i [°]	Ω [°]	ω [°]	T_0 [s]	RMS [″]
2.99	$1.27{\cdot}10^{5}$	$2.32{\cdot}10^{-3}$	$2.52{\cdot}10^{-3}$	$2.09{\cdot}10^{-2}$	12.84	$3.06{\cdot}10^{3}$	0.47

CRAO and ESASDT

Δt [h]	a [m]	e	i [°]	Ω [°]	ω [°]	T_0 [s]	RMS [″]
0.11	$4.64{\cdot}10^{4}$	$4.24{\cdot}10^{-3}$	$1.92{\cdot}10^{-2}$	0.11	65.34	$1.55{\cdot}10^{4}$	0.29
2.99	$3.78{\cdot}10^{5}$	$7.20{\cdot}10^{-3}$	$4.67{\cdot}10^{-3}$	$5.61{\cdot}10^{-2}$	7.78	$2.10{\cdot}10^{3}$	0.60

Table 10: Formal errors for the elliptical orbit determination of object GEO_M3 representing two tracks observed with the ZIMLAT (top) and two tracks observed with the ZIMLAT and the ESASDT (bottom).

ZIMLAT

Δt [h]	a [m]	e	i [°]	Ω [°]	ω [°]	T_0 [s]	RMS [″]
0.94	$2.03{\cdot}10^{6}$	$2.18{\cdot}10^{-2}$	$3.06{\cdot}10^{-2}$	0.32	20.95	$2.94{\cdot}10^{3}$	0.54
7.95	$4.21{\cdot}10^{4}$	$8.89{\cdot}10^{-4}$	$2.00{\cdot}10^{-3}$	$1.55{\cdot}10^{-2}$	0.26	64.92	0.51

ZIMLAT and ESASDT

Δt [h]	a [m]	e	i [°]	Ω [°]	ω [°]	T_0 [s]	RMS [″]
0.08	$2.10{\cdot}10^{5}$	$1.53{\cdot}10^{-2}$	$8.76{\cdot}10^{-2}$	$4.73{\cdot}10^{-2}$	4.02	$1.27{\cdot}10^{3}$	0.55
0.94	$1.98{\cdot}10^{5}$	$1.14{\cdot}10^{-3}$	$7.65{\cdot}10^{-3}$	$2.92{\cdot}10^{-2}$	2.62	$4.86{\cdot}10^{2}$	0.18
7.95	$1.06{\cdot}10^{4}$	$2.39{\cdot}10^{-4}$	$4.62{\cdot}10^{-4}$	$6.27{\cdot}10^{-3}$	$4.79{\cdot}10^{-2}$	19.11	0.09

tor of about $4-5$ smaller than those from one site. This corresponds to the ratio of the RMS values.

Conclusions

Observations from multiple sites clearly improve the determined orbit compared to orbits established with only one site for arcs not longer than a few hours. Simultaneous observations allow determining all six orbital elements, which is not the case when a short arc is observed from one site only.

When observing from two sites, the accuracy of the orbits depends on the separations in longitude and latitude between the two sites. The larger the separation, the more the orbit quality improves. If the separation of the positions is mainly in longitude, follow-up observations after one hour lead to a better result than simultaneous observations. In the other case, a larger separation in latitude, better results can be achieved with simultaneous observations.

In which case should simultaneous observations from two or three sites be preferred to observations from one site? Simultaneous observations are helpful if the emphasis is on the determination of six parameters, e.g., for statistical analysis. The obtained orbits, however, do not guarantee a recovery during the following night, which is a requirement to maintain a catalogue. Additional follow-up tracks are still required. For this purpose, simultaneous observations can therefore not be recommended for several reasons: 1) The total observation time needed will be larger than with the follow-up strategy, as the telescopes are almost fully used for surveying the same selected field and can not be used for other tasks. At least one additional telescope is needed to perform the necessary follow-up observations. 2) Depending on the distance between the sites the observable part of the GEO belt will be smaller than in the case of using one site. 3) The weather conditions have to be clear at all involved sites.

A second site is useful if it is used to perform follow-up observations, although the number of follow-up tracks needed to recover the object during the following night is not reduced. The size of the needed FOV is, however, clearly reduced. Furthermore, the orbit determined from the observations of the first night is more accurate.

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